

THE METALS AND MINERAL FUELS CRISIS

Facts and Predictions

**Richard A. Arndt
and
L. David Roper**

*Department of Physics
Virginia Polytechnic Institute
and State University
Blacksburg, Virginia 24060*

TABLE OF CONTENTS

Tables		iii
Figures		iv
Preface		v
CHAPTER ONE:	Introduction	1
CHAPTER TWO:	Depletion Theory	3
CHAPTER THREE:	Mineral Production Data	9
CHAPTER FOUR:	Production Data Fitting Procedure	11
CHAPTER FIVE:	Fits to United States and World Production Data	13
	I. Metals	
	Antimony 14, Arsenic, white 16, Bauxite 18, Beryl 20, Bismuth 22, Cadmium 24, Chromite 26, Cobalt 28, Copper 30, Gold 32, Iron Ore 34, Lead 36, Magnesium 38, Manganese Ore 40, Mercury 42, Molybdenum 44, Nickel 46, Niobium-Tantalum 48, Platinum Group 50, Selenium 52, Silver 54, Tellurium 56, Tin 58, Titanium, Ilmenite 60, Titanium, rutile 62, Tungsten Ore 64, Uranium Oxide 66, Vanadium 68, Zinc 70.	
	II. Mineral Fuels	
	Crude Oil 72, Natural Gas 74, Coal 76, Mineral Fuels 78, Peat 78.	
	III. Selected Nonmetals	
	Mica Sheet 80, Phosphate Rock 80.	
CHAPTER SIX:	Conclusion	83
References		88
Index		89

TABLES

Table 1.	Production Data Conversion Factors	10
Table 2.	Mineral Depletion Parameters	82
Table 3.	Depletion Half Dates	84
Table 4.	Asymmetry Categories	85

FIGURES

PREFACE

Figure No.	Description	Page	Figure No.	Description	Page
1	U.S. Antimony data and fit.	15	32	U.S. Niobium-Tantalum data and fit.	49
2	World Antimony data and fit.	15	33	World Niobium-Tantalum data and fit.	49
3	World Arsenic (white) data and fit.	17	34	U.S. Platinum Group data and fit.	51
4	U.S. Bauxite data and fit.	19	35	World Platinum Group data and fit.	51
5	World Bauxite data and fit.	19	36	U.S. Selenium data and fit.	53
6	U.S. Beryl data and fit.	21	37	U.S. Silver data and fit.	55
7	World Beryl data and fit.	21	38	World Silver data and fit.	55
8	World Bismuth data and fit.	23	39	U.S. Tellurium data and fit.	57
9	U.S. Cadmium data and fit.	25	40	U.S. Tin data and fit.	59
10	World Cadmium data and fit.	25	41	World Tin data and fit.	59
11	U.S. Chromite data and fit.	27	42	U.S. Titanium (ilmenite) data and fit.	61
12	World Chromite data and fit.	27	43	World Titanium (ilmenite) data and fit.	61
13	World Cobalt data and fit.	29	44	U.S. Titanium (rutile) data and fit.	63
14	U.S. Copper data and fit.	31	45	World Titanium (rutile) data and fit.	63
15	World Copper data and fit.	31	46	U.S. Tungsten Ore data and fit.	65
16	U.S. Gold data and fit.	33	47	World Tungsten Ore data and fit.	65
17	World Gold data and fit.	33	48	U.S. Vanadium data and fit.	69
18	U.S. Iron Ore data and fit.	35	49	World Vanadium data and fit.	69
19	World Iron Ore data and fit.	35	50	U.S. Zinc data and fit.	71
20	U.S. Lead data and fit.	37	51	World Zinc data and fit.	71
21	World Lead data and fit.	37	52	U.S. Crude Oil data and fit.	73
22	U.S. Magnesium data and fit.	39	53	World Crude Oil data and fit.	73
23	World Magnesium data and fit.	39	54	U.S. Natural Gas data and fit.	75
24	U.S. Manganese Ore data and fit.	41	55	World Natural Gas data and fit.	75
25	World Manganese Ore data and fit.	41	56	U.S. Coal data and fit.	77
26	U.S. Mercury data and fit.	43	57	World Coal data and fit.	77
27	World Mercury data and fit.	43	58	U.S. Mineral Fuels data and fit.	79
28	U.S. Molybdenum data and fit.	45	59	U.S. Peat data and fit.	79
29	World Molybdenum data and fit.	45	60	U.S. Mica Sheet data and fit.	81
30	U.S. Nickel data and fit.	47	61	U.S. Phosphate Rock data and fit.	81
31	World Nickel data and fit.	47	62	U.S. Iron Ore fits for symmetry (n=1) and asymmetry (n=5).	86

The "oil crisis" of the early 1970's has led to a desire for information concerning the depletion status of other nonrenewable resources, for example, the metals. In this monograph we attempt to provide a tentative measure of the depletion status of mineral fuels, metals, and a few nonmetals for the United States and the World.

Herein we predict future production for a specific nonrenewable resource by fitting the past production data to various functions that have, at least, gross features that are expected to occur in such production data. For a mineral that is far past its production peak (e.g., platinum and mercury in the United States), the projection into the future should be fairly accurate. For production data that have not yet peaked (e.g., oil and gas) the prediction is shaky — it probably is too pessimistic, for reasons that are explained in this monograph.

However, such pessimistic predictions are still useful in planning for the future: One could argue that it is better to be pessimistic than optimistic for planning purposes. It is hoped that this study will be useful in the continuation of recent pioneering attempts to use systems analysis to project the future of the World or regional areas of the World, the best known of which are reported in Meadows, et al., Cole, et al., and Mesarovic and Pestal.¹

From our study we conclude that the World, and especially the United States, is in a more severe "metals crisis" than "energy crisis." That is, many metals have already peaked or will soon peak in production, whereas no mineral fuels have peaked yet. However, the crucial importance of energy to all motions and transformations of matter, including mining and processing of metals, and the lack of a wide variety of mineral fuels and other presently available energy substitutes make the early stages of an energy crisis much more traumatic than similar stages of a metals crisis.

There is no lesson from history to make it appear inevitable that we will be able to easily obtain minerals from some new, as yet untapped, source when the land source effectively disappears. We may have a long period of extensive recycling, but the second law of thermodynamics mandates that recycling is necessarily less than perfect. So new sources must eventually be found elsewhere than the land if we are to even maintain

¹Meadows, et al., *The Limits to Growth* (Universe Books); Cole, et al., *Models of Doom* (Universe Books); and Mesarovic and Pestel, *Mankind at the Turning Point* (E.P. Dutton & Co., Inc.).

industrial production levels near those now in existence. This means that eventually energy efficient and environmentally nondestructive processes must be developed to extract minerals from the oceans, the moon, the planets, or the asteroids.²

The authors are grateful for the help in collecting the data by Dr. Selim Sancaktar and for the constant encouragement and help of Dr. Madan L. Gupta.

L. David Roper

Richard A. Arndt

²The mathematical details of the theory involved and the data used in carrying out this research is being published as *Depletion of United States and World Mineral Resources* (tentative publication date, January, 1977). An interpretation and simplification for a general audience has been written by one of us (LDR) and is available as *Where Have All the Metals Gone?*. (Both books are published by University Publications, P. O. Box 47, Blacksburg, Virginia 24060.)

Chapter

1

INTRODUCTION

In recent years many people have made estimates of the nonrenewable mineral resources available for use by man. Some have made the unrealistic assumptions that either the production rate of a specific mineral remains constant at the current rate (e.g., see Cloud, 1973) or the production rate *increase* remains constant at its current rate (e.g., see Gabel, 1975). Others (e.g., see Lasky, 1951; Lasky, 1955; and Hubbert, 1969) have realistically tried to allow for the fact that at some point in time the production rate must peak and thereafter decrease, on the average, until essentially all of the available amount of that mineral has been extracted from the earth.

By "available amount" is meant the amount that man finally deems worthwhile to extract. As the energy, material investment, and environmental degradation for further extraction of a given mineral increases, society gradually decides that there are other pursuits more worth the expenditure even though there still may be huge amounts of that mineral left somewhere in the earth's crust. At some point, for example, recycling of the mineral or substitution of some other resource (nonrenewable or renewable) becomes more desirable than further extraction. See Lasky (Lasky, 1951 and 1955) and Cook (Cook, 1975) for more detailed discussions of this. (Two other interesting discussions of minerals depletion are by Steidle (Steidle, 1952) and Park and Freeman (Park and Freeman, 1968).)

In this monograph we attempt to improve the realistic approach and extend it to all metals and mineral fuels for both the United States and the World where sufficient mineral production data are available. The World data are less copious, less reliable, and much less near depletion than are the United States data.

Our approach is to find a function or functions for $Q(t)$, the amount of the mineral that remains to be extracted at time t , by fitting the production data for the mineral to $P(t) = -dQ(t)/dt$ by varying parameters in the $Q(t)$ function(s). The best fit thereby obtained can be used to predict future production of that mineral. This prediction method is called the "production-history projections" method (Cook, 1975). The best known proponent of this method is Hubbert for crude oil and natural gas (Hubbert, 1969). However, our work differs from Hubbert's in that we allow the data to "choose" which of several functions are best and we do not use any data other than production data. Other data, such as discovery rates and reserves estimates, can then be used as checks on our results.

There are other depletion prediction methods. Cook (Cook, 1975) ably discusses the different methods in detail but concludes:

The production-history method of forecasting depletion may be a better guide to national policy than are the geologic-economic methods, for it yields a direct and continuous forecast of supply rates. In addition, its errors will fall on the side of prudence rather than on the side of flatulent optimism.

We find it convenient to condense our future predictions in terms of (1) the percent already extracted by 1975, (2) the date when production peaks (t_p = peak date), (3) the date when the mineral is one-half extracted ($t_{1/2}$ = half date), and (4) the date when the mineral is three-fourths extracted ($t_{3/4}$ = three-quarter date). For the reader who wants a quick answer, Table 2 and Table 3 in Chapter Six give these numbers for various minerals.

We do not expect this approach to give a very precise prediction for those minerals which have not yet reached their production peak. We expect that our prediction is pessimistic in these cases, which include about one-fourth of the United States minerals that we study and about three-fourths of the World minerals that we study. In Chapter Six we shall present a possible way to improve the prediction for minerals that have not yet peaked.

Our findings can be succinctly stated as follows:

1. The United States is in a much more severe "metals crisis" than "fuels crisis." That is, most metals have already peaked in production, whereas no mineral fuel has yet peaked (although crude oil and natural gas are close). However, the crucial importance of energy to all motions and transformations of matter, including mining and processing of metals, and the lack of a wide variety of mineral fuels and other presently available energy substitutes make the early stages of an energy crisis much more traumatic than similar stages of a metals crisis.
2. The World's mineral crisis will be about twenty years after the United States' mineral crisis. Rough dates for crisis recognition are 1970 for the United States and 1990 for the World.

Chapter 2 DEPLETION THEORY

We present here an outline of a theory of depletion of nonrenewable resources. More mathematical details are given in our more complete account of this work.³

We start from two basic irrefutable facts:

1. The earth (or any portion of the earth) is a finite source of any mineral.
2. As a mineral is extracted from the earth it becomes steadily more difficult to extract the remainder. By "more difficult" is meant that more materials and energy are required and more environmental degradation occurs.

These two facts define complicated nonlinear interactions among all minerals: The increasing scarcity with time of one mineral (say crude oil) makes it more difficult to obtain another mineral (say iron ore) which may be crucially important in the extraction of the first mineral. This is only one of scores of nonlinear interactions that are simultaneously at work.

Common sense tells one the kind of long-term "average" production rate behavior to expect for any mineral. There are components of both technology and sociology that interplay in the behavior.

1. In the *earliest stage* the mineral is relatively readily available, but the technology for its extraction and society's need for it are undeveloped. Therefore, the production rate will increase slowly at first. However, as the extracted mineral enters into the mainstream of the society its presence will generate more need for it and thereby generate more advanced extraction technology. Thus, it is reasonable to assume that the production rate at earliest times will be some increasing function of the amount already extracted at that time. (Let Q_∞ = amount that will be eventually extracted and $Q(t)$ the amount left to be extracted at time t ; then the production rate $P = -dQ/dt$ is some function of $(Q_\infty - Q)$.) Since any smooth function can be expanded as a power series in an independent variable, at the very earliest times P should be proportional to some power of $(Q_\infty - Q)$. The simplest assumption and one that often works in other similar situations is that $P \propto (Q_\infty - Q)$ at the earliest times. However, we shall consider more complicated possibilities.

³Arndt and Roper, *Depletion of United States and World Mineral Resources*, University Publications, P. O. Box 47, Blacksburg, Virginia 24060.

2. At the *latest stage* when the mineral is almost completely depleted, the principal limitation on the production rate P will be the amount left to be extracted $Q(t)$ at that time. Again, at the very latest times P should be proportional to some power of Q . The simplest assumption and one that often works in other similar situations is that $P \propto Q$ at the latest times. However, we shall consider more complicated possibilities.
3. At *intermediate times* there are no rational arguments that we can muster for any particular functional form for P as a function of Q . So we shall consider several possibilities and let the production data for a given mineral "choose" which of the possibilities works best by performing least-square fits to the data. Some obvious statements can be made, however: After rising slowly at earliest times, the production rate should begin to accelerate, then later (at an inflection point) decelerate until the production rate peaks at some time. Then the rate will begin to decline in a similar, but not necessarily symmetrical, fashion. Finally, P will asymptotically approach zero. The simplest assumption that one could make which yields this kind of behavior is that P is strictly proportional to the first power of both $(Q_\infty - Q)$ and Q at all times; i.e.,

$$P = - \frac{dQ}{dt} = kQ (Q_\infty - Q) \quad (1)$$

where k is a rate constant that is a measure of the usefulness of the mineral and the long-term economic conditions of the society. (One can define a time constant $\tau \equiv 1/k$.) We discuss this equation, as well as more complicated cases, below.

Of course, in reality for a particular mineral the long-term average behavior described above will not precisely describe the production-rate behavior. There are short-term social phenomena, such as wars and economic depressions, that can and sometimes do cause rather large fluctuations in the production rate. (A detailed study of the correlations of these mineral-production fluctuations with specific social phenomena would be interesting. We do not attempt it here.) These short-term fluctuations exhibit behavior similar to that described above for the long-term average behavior except that the rate constant k is greatly increased (time constant τ is greatly reduced). (We shall often refer to the long-term average behavior as the "background" behavior.) There are two situations that could exist:

1. The short-term fluctuations have little or no effect on the long-term background behavior. That is, the rate constant for the background behavior is unchanged as short-term fluctuations occur.
2. The short-term fluctuations are evidence of changes in the long-term use of that mineral either because of the onset of new long-term social phenomena or new mineral technology (e.g., substitution of another mineral for it in its major use). That is, the rate constant for the background behavior is changed as short-term fluctuations occur.

Of course, it is possible that the long-term background rate "constant" k is not really a constant in time even in the absence of fluctuations. In fact, one would think that after a mineral has become an integral part of a society's economy, the society will make

a large effort to keep its production rate up when it otherwise would decline sharply for constant k . That is, the society's increased efforts to extract the mineral will cause k to decrease with time rather than be constant. This will cause the production-rate curve to be asymmetrically skewed toward large times; we shall later see that most nearly depleted United States minerals have such skewed production-rate curves. Also, gradual substitution of one mineral for another (e.g., oil for coal) could cause k to change with time.

A. LOGISTIC CURVE

Eq. (1) given above for the production rate P as a function of the amount $Q(t)$ yet to be extracted at time t has as its solution the widely-used *logistic curve*:

$$Q(t) = \frac{1}{2} Q_\infty \left[1 - \tanh \left(\frac{t - t_{1/2}}{2\tau} \right) \right] \quad (2)$$

where $\tau \equiv 1/k$ and $t_{1/2}$ is the time at which the mineral is one-half depleted. We shall label $t_{1/2}$ the "half-date."

B. VARIABLE DECAY-RATE MODEL

One can complicate the simple model developed above by assuming that the decay rate, $k(t)$, is a function of time. Then

$$P = - \frac{dQ}{dt} = k(t) \frac{Q}{Q_\infty} (Q_\infty - Q), \quad (3)$$

which has the solution

$$Q(t) = \frac{1}{2} Q_\infty \left[1 - \tanh \left(\frac{g(t) - g(t_{1/2})}{2} \right) \right] \quad (4)$$

where

$$g(t) \equiv \int_0^t k(t) dt.$$

This approach could be used to give the large-time skewing that often occurs in nearly depleted production data.

C. GENERALIZED VERHULST CURVE

A more complicated, but still analytically solvable, case that contains A. and B. above as special cases is the Verhulst equation

$$P = - \frac{dQ}{dt} = \frac{k(t)Q}{n} \left[1 - \left(\frac{Q}{Q_\infty} \right)^n \right], \quad (5)$$

which has the solution

$$Q(t) = \frac{Q_\infty}{\left[1 - (1 - 2^n) e^{(g(t) - g(t_{1/2}))}\right]^{1/n}} \quad (6)$$

$\xrightarrow[k = \text{constant} = 1/\tau]$

$$\frac{Q_\infty}{\left[1 - (1 - 2^n) e^{\left(\frac{t - t_{1/2}}{\tau}\right)}\right]^{1/n}}$$

Note that this approach assumes that P is linear in Q for large times but is nonlinear in $(Q_\infty - Q)$ for small times.

For $n = 1$, this equation can be shown to be the same as Eq. (4) above, which is the same as Eq. (2) when $k(t)$ is not a function of t . However, even with k a constant Eq. (6) contains a large-time skewing if $n > 1$. (The generalized Verhulst curve is skewed toward short times for $0 \leq n < 1$, is symmetrical for $n = 1$, and is skewed toward large times for $n > 1$.) We choose this approach in fitting the nearly depleted skewed data rather than the approach given in B. above, because it is mathematically simpler.

D. GOMPERTZ CURVE

Another special case of the Verhulst curve is when k is a constant and $n = 0$. This yields the Gompertz Curve which is given by the equations

$$P = -\frac{dQ}{dt} = \frac{1}{\tau} Q \ln\left(\frac{Q}{Q_\infty}\right) \quad (7)$$

and

$$Q(t) = Q_\infty \left(\frac{1}{2}\right)^{\exp[(t-t_{1/2})/\tau]} \quad (8)$$

This is a curve that is skewed toward short times. We will rarely have occasion to use Eq. (8) in our later fits to the data.

E. ERROR FUNCTION CURVE

There are at least two other symmetric peaked functions that often occur in nature. One is the Gaussian function:

$$P = -\frac{dQ}{dt} = \frac{Q_\infty}{\tau\sqrt{\pi}} \exp\left[-\left(\frac{t - t_{1/2}}{\tau}\right)^2\right] \quad (9)$$

or

$$Q(t) = \frac{1}{2}Q_\infty \left[1 - \operatorname{erf}\left(\frac{t - t_{1/2}}{\tau}\right)\right] \quad (10)$$

F. INVERSE COTANGENT CURVE

Another symmetric peaked function that often occurs in nature is the Lorentzian function:

$$P = -\frac{dQ}{dt} = \frac{Q_\infty}{\pi\tau} \left[\frac{1}{1 + \left(\frac{t - t_{1/2}}{\tau}\right)^2} \right] \quad (11)$$

or

$$Q(t) = \frac{1}{\pi} Q_\infty \operatorname{ctn}^{-1}\left(\frac{t - t_{1/2}}{\tau}\right) \quad (12)$$

One can show that this is a situation in which P is quadratic in $(Q_\infty - Q)$ for short times and in Q for long times but is not proportional to $Q^2(Q_\infty - Q)^2$ for intermediate times.

MINERAL PRODUCTION DATA

The major sources for United States and World mineral production data is *Minerals Yearbook*, (U. S., 1932-1972) and its predecessor, *Mineral Resources in the United States* (U. S., 1883-1931). In Chapter Five we shall refer to both of these as "Minerals Yearbook." We obtained United States data for 1973 and 1974 from "Mineral Industries Surveys" (Bureau of Mines, 1973-74). Where other sources of data are used they are indicated in Chapter Five for each mineral. Our more complete account of this work⁴ contains tables of the data used in our fits.

Occasionally the data are difficult to interpret. The World data are often very unreliable because some countries' data are not included. We found no World data for 1973 and 1974, except crude oil and natural gas for 1973. Usually the World data do not extend as far back as do the United States data.

Data for some years are given in terms of ore mass and for some years in terms of the metal content of the ore for a given mineral. We always convert all data of a mineral to either one or the other by using overlapping data of the two types to establish an approximate ratio. Where this is done is indicated in Chapter Five for each mineral.

The units used are occasionally different for different dates for a given mineral. We almost always convert all past data to the units most recently used. Some necessary conversion factors are given in Table 1.

It would be very helpful for future work of this type if persons knowledgeable in the various minerals would convert all past data into compatible form.

Although we use only production data in making our predictions, it is useful to have mineral-reserves estimates at different dates to compare to our $Q(t)$ values. One should realize, however, that "It is not at all certain that ultimate recovery will extend to the limit of possible reserves" (Cook, 1975). Also, there are many methods for calculating reserves that yield vastly different numbers. The most complete set that we could find of United States and World reserves estimates is given by Frasché (Frasché, 1962) for 1960. Where other reserves values are used is indicated in Chapter Five for each mineral.

⁴Arndt and Roper, *Depletion of United States and World Mineral Resources*, University Publications, P. O. Box 47, Blacksburg, Virginia 24060.

TABLE 1
Production Data Conversion Factors

From	To long ton	short ton	metric ton	lb	kg	ft ³	m ³
long ton	1	1.12	1.01605	2240			
short ton	0.89286	1	0.90718	2000			
metric ton	0.98420	1.10232	1	2204.6	1000		
lb	4.464x10 ⁻⁴	5x10 ⁻⁴	4.53x10 ⁻⁴	1	0.4536		
ft ³						1	0.028317
m ³						35.3144	1
TO	37.20 x 10 ⁻⁵	4.160 x 10 ⁻⁵	3.780 x 10 ⁻⁵	0.08333			
FL	0.0339	0.0380	0.0345				
RJ			0.001	2.2046			26.456

From	To kWh
barrels of oil	1640
ft ³ of natural gas	0.293
short tons of coal	7325

2 Troy oz = 1 lb.
 1 short ton = 2000 lb = 26,456 TO
 1 metric ton = 1.10232 ST = 26,456 TO
 (FL)
 26 lb flask:
 1 short ton = 2000 lb = 26,32 FL
 1 m³ = 1.10232 ST = 29.01 FL

Chapter

4

PRODUCTION DATA FITTING PROCEDURE

We want to achieve a good fit to the production data for a mineral using the functions given previously and then use the fit to predict the production rate in the future. Our procedure is to fit each mineral's data with the four functions given in Eqs. (2), (8), (10), and (12). For a mineral that has not yet peaked in production we use only the symmetric functions of Eqs. (2), (10), and (12). In some cases where the transient fluctuations are large we fit the production data with a sum of peaked functions. If a mineral is past its production peak we generalize the fit to Eq. (2) by allowing n to move away from n = 1 in a fit to Eq. (6). This allows for an asymmetric peak (n ≠ 1).

The fitting is accomplished by a least-squares-fit computer code developed by one of the authors (R.A.A.). It is necessary in such a fitting to have "errors" for the data to be fitted. We manufacture errors for the data points such that at least one of the four functions of Eqs. (2), (8), (10), and (12) has a reasonable probability of good fit (0.5 to 1). These errors then represent the average deviation of the transient fluctuations from the background smooth behavior. They are shown in the various production-rate figures in Chapter Five. A least-squares fit generates an error matrix for the variable parameters which can be used to calculate an error channel for any function of the parameters. In the Q(t) figures in Chapter Five we shall show the error channels as light curves flanking the dark Q(t) function curve.

Peaked function fits to data that have not yet peaked are very sensitive to the initial value used for t_{1/2}; a fit often gets lodged in a local χ² (the measure of good least-squares fit) minimum rather than finding the lowest minimum. To obviate this we fix t_{1/2} at various values spaced at five year intervals over a large time span and then vary all other parameters. Often we thus found the lowest minimum at a different t_{1/2} value than the value we had determined by varying all parameters simultaneously.

**FITS TO UNITED STATES
AND WORLD
PRODUCTION DATA**

We present below, in alphabetical order, the fits to the United States and World metals and mineral-fuels production data, and to two United States nonmetal minerals as examples of possible future work. In the graphs of the production-rate data presented below we often do not plot the data for every year because they are too close together.

The fit parameters are given in our more complete account of this work.⁵ For the reader who wants a quick answer, Table 2 and Table 3 in Chapter Six give some of the depletion parameters for the minerals discussed in this chapter. Perhaps other readers would also benefit by quickly perusing Tables 2 and 3 before studying the details for each mineral in this chapter.

⁵Arndt and Roper, *Depletion of United States and World Mineral Resources*, University Publications, P. O. Box 47, Blacksburg, Virginia 24060.

I. METALS

ANTIMONY

United States: Data from 1932 to 1952 in *Minerals Yearbook* are given in terms of antimony ore and concentrate, whereas after 1952 they are in terms of antimony content. Some years in which data are available in both forms allow one to establish a ratio of approximately three between the ore and the antimony content. This ratio was used to convert all data to antimony content. It is difficult to figure out what the data from 1880 to 1931 represent. Therefore, we do not use any of the data prior to 1932.

The huge increase (factor of six) in antimony production during World War II indicates that there may be special occasions in the future when urgent needs will cause large transient production peaks. With that caveat, our fit indicates that we are past the peak in antimony production. Of course, the discovery of some new, important use for antimony could drastically change the situation. This is a very uncertain prediction.

The 1960 reserves estimate (●) (Frasché, 1962) is somewhat less than our Q(1960) value.

**Peak date — 2000, Half date — 1978,
48% gone in 1975.**

World: Prior to 1912 the data are for mixed ore and metal. Minor fluctuations occur on a rather smooth rise to a production peak at about 2020. As for fits to all production data that have not yet peaked, we must caution that our prediction may be pessimistic since it does not allow for any asymmetry. However, the 1960 reserves estimate (●) (Frasché, 1962) is much smaller than our Q(1960) value.

**Peak date — Half date — 2021,
20% gone in 1975.**

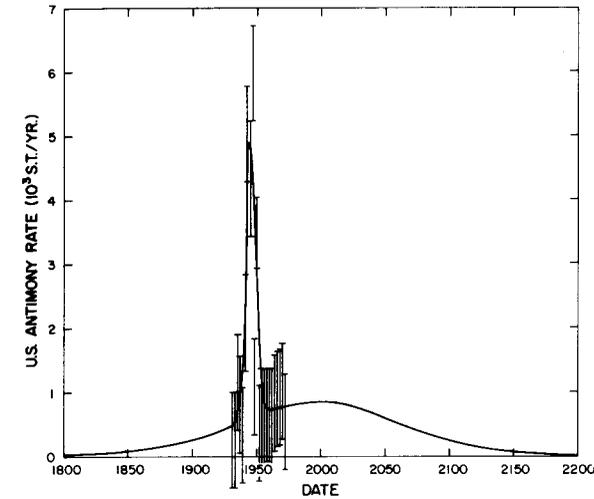


Figure 1

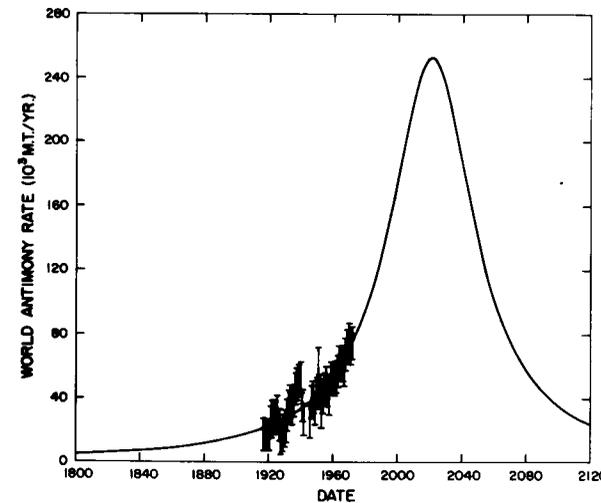
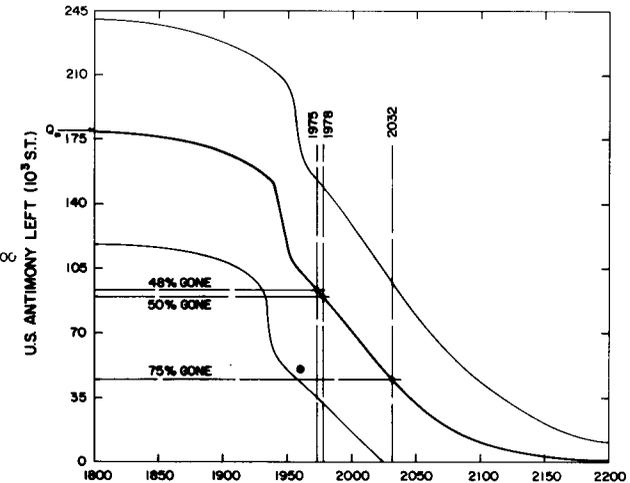
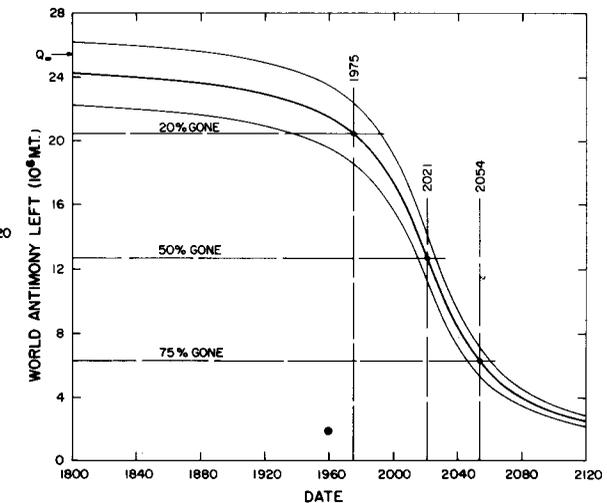


Figure 2



ARSENIC, WHITE

United States: Data for only ten years (1939 and 1951-1959) are available in *Minerals Yearbook*, which are not enough to obtain a meaningful fit. They indicate a rapidly falling production rate.

World: We found no data prior to 1913 or for the years 1948-1950. This is one of the few World minerals that appear to have already peaked. We predict a large asymmetry (solid curve) as the World tries hard to maintain white arsenic production near the rate to which it has become accustomed. The dashed curve is a symmetric fit to the data, which is a slightly poorer fit than the asymmetric fit.

**Peak date — 1934, Half date — 1992,
41% gone in 1975.**

The available U.S. White Arsenic production data are not sufficient to make a fit.

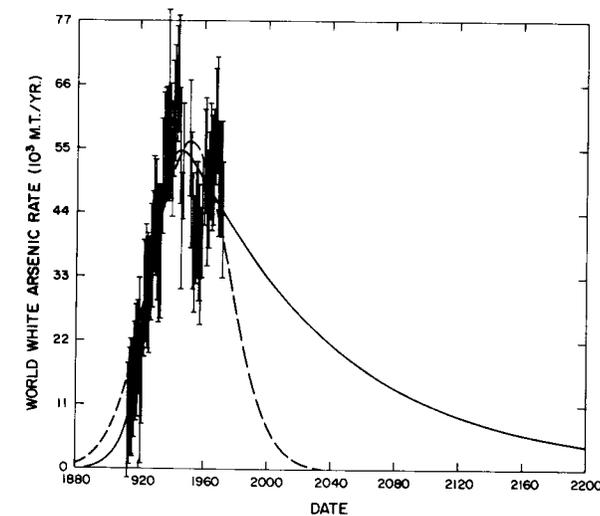
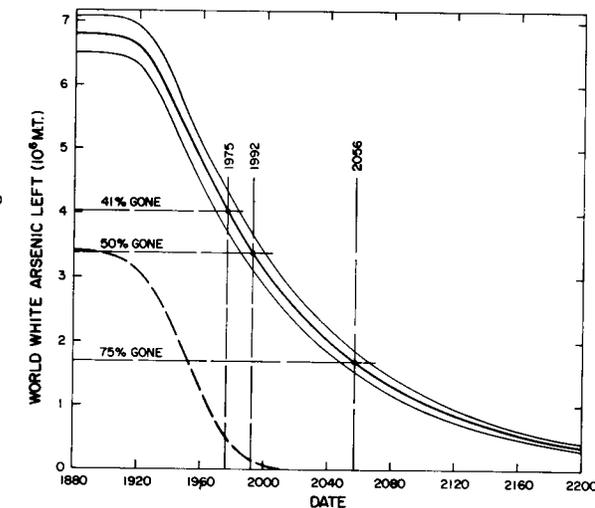


Figure 3



BAUXITE

United States: The huge increase (factor of seven) in bauxite production during World War II indicates that there may be special occasions in the future when urgent demand will cause large transient production peaks. With that caveat, our fit indicates that we are just barely past the peak in bauxite production. The 1960 reserves estimate (●) (Frasché, 1962) is slightly less than our Q(1960) value.

**Peak date — 1968, Half date — 1966,
66% gone in 1975.**

World: World War II is only a small blip on a fast rise in bauxite production. Our *very uncertain* prediction is a peak around 1993. As for all fits to production data that have not yet peaked, we must caution that our prediction may be pessimistic since it does not allow for any asymmetry. However, the 1960 reserves estimate (●) (Frasché, 1962) is somewhat less than our Q(1960) value.

**Peak date — Half date — 1993,
15% gone in 1975.**

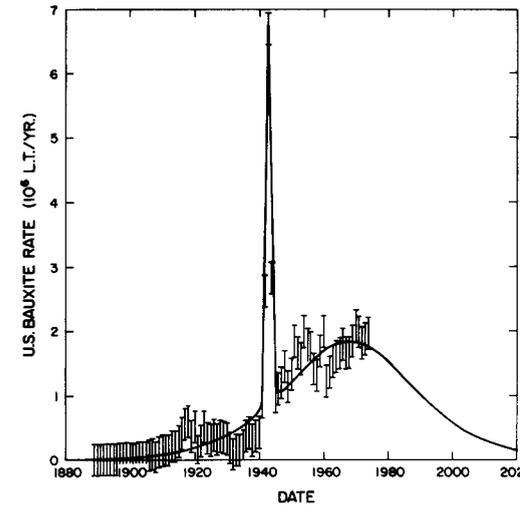


Figure 4

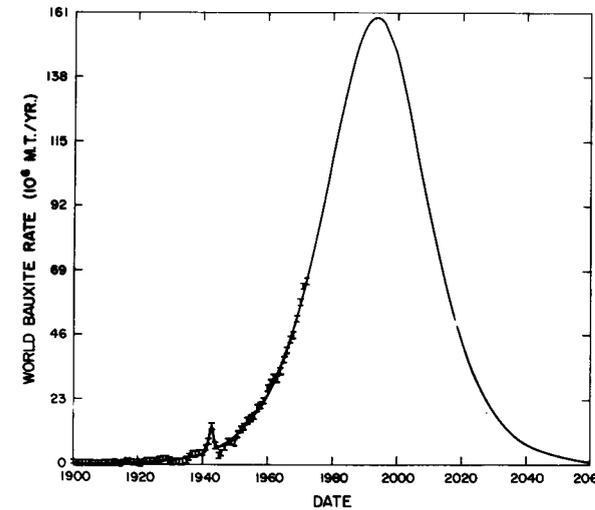
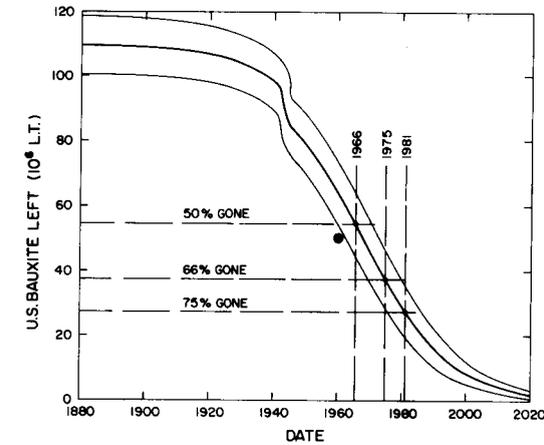
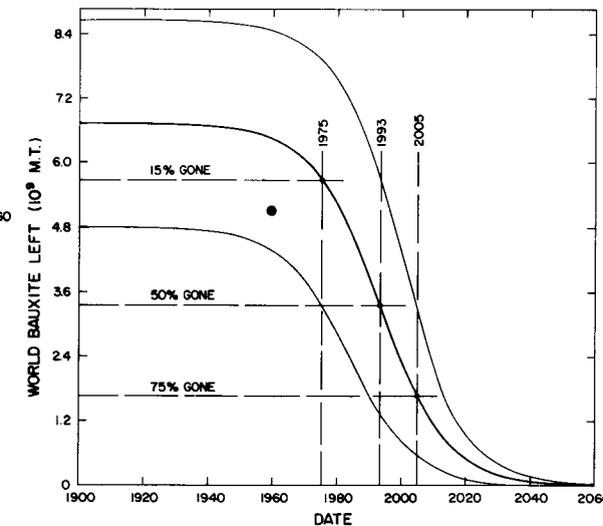


Figure 5



BERYL

United States: Data are missing in *Minerals Yearbook* for the years 1964-1967 and 1969-1974, and we found no data prior to 1935. A Gompertz curve (solid curve) is the best fit to the production data because of the isolated low point for 1968. However, we choose the logistic function (dashed curve) for our prediction because it gives almost as good a fit as the Gompertz function, it is more optimistic, and is closer to the 1960 reserves estimate (●) (Frasché, 1962). According to Brobst and Pratt (Brobst and Pratt, 1973) a new type of ore is beginning to be mined in the United States, so that our prediction should probably be ignored.

**Peak date — Half date — 1960,
93% gone in 1975.**

World: We found no data prior to 1935. This is one of the few World minerals that appear to have already peaked. There is no measurable asymmetry up to the present time. The 1960 reserves estimate (●) (Frasché, 1962) is somewhat higher than our Q(1960) value.

**Peak date — Half date — 1959,
84% gone in 1975.**

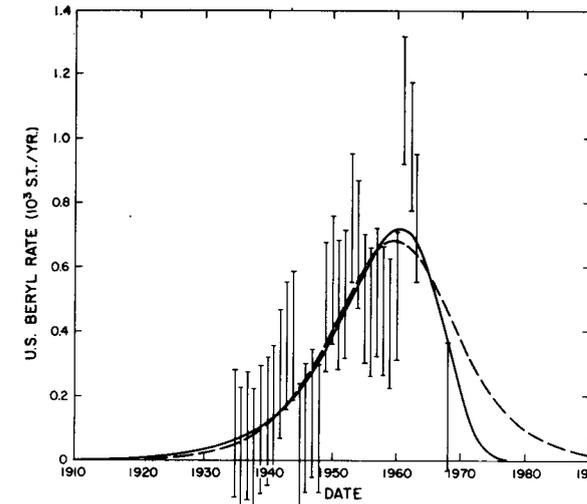


Figure 6

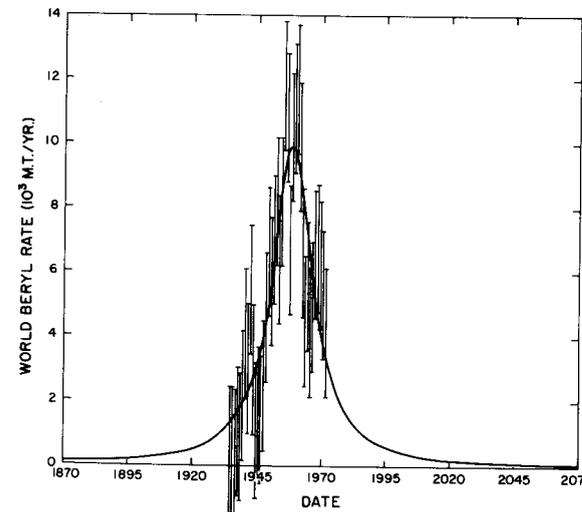
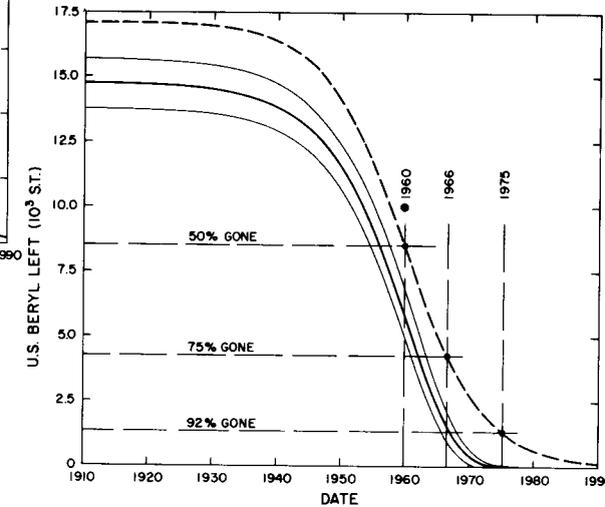
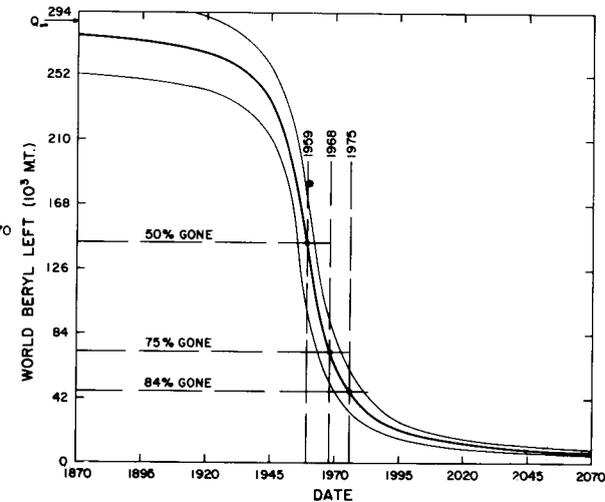


Figure 7



BISMUTH

United States: We did not locate any bismuth production data.

World: We found no data prior to 1938. According to our fit, World bismuth production may peak very soon, although there is a large uncertainty. As for all fits to production data that have not yet peaked, we must caution that our prediction may be pessimistic since it does not allow for any asymmetry. However, the 1960 reserves estimate (●) (Frasché, 1962) is much smaller than our Q(1960) value.

Peak date — Half date — 1983,

36% gone in 1975.

No U. S. Bismuth production data could be located.

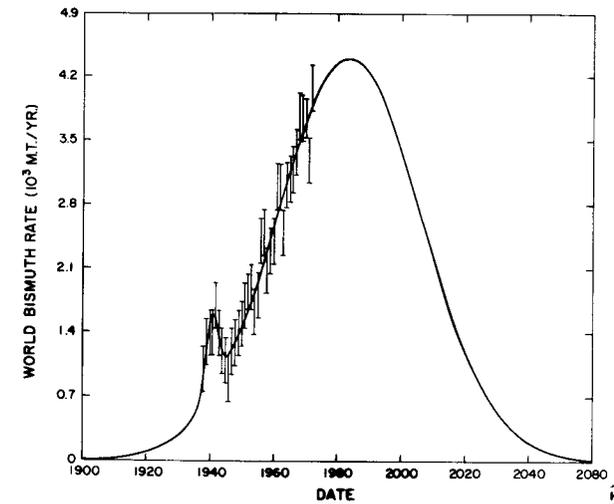
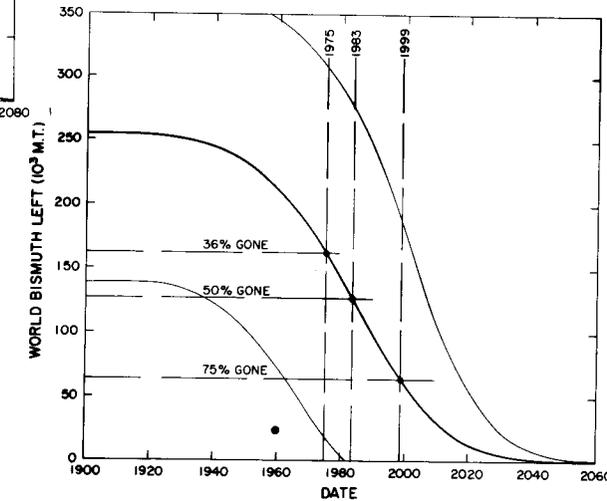


Figure 8



CADMIUM

United States: We found no data for 1949 and 1950. It appears unlikely that the production peak is a short term effect, although that is possible. The asymmetric fit (solid curve) is not quite as good a fit as is the symmetric fit (dashed curve); however, we choose the asymmetric fit for our prediction because it is more optimistic. The 1960 reserves estimate (●) (Frasché, 1962) is a factor of four lower than our Q(1960) value.

**Peak date — 1957, Half date — 1972,
54% gone in 1975.**

World: We found no data prior to 1928. Our prediction is that production peaks in the near future (~1980). As for all fits to production data that have not yet peaked, we must caution that our prediction may be pessimistic since it does not allow for any asymmetry. However, the 1960 reserves estimate (●) (Frasche, 1962) is a factor of two lower than our Q(1960) value.

**Peak date — Half date — 1978,
44% gone in 1975.**

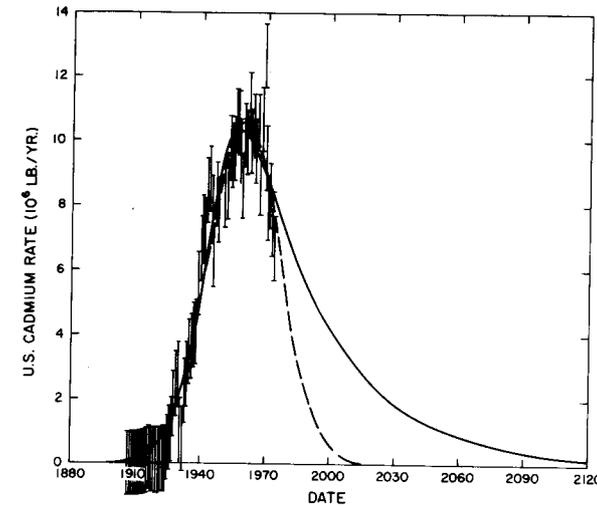


Figure 9

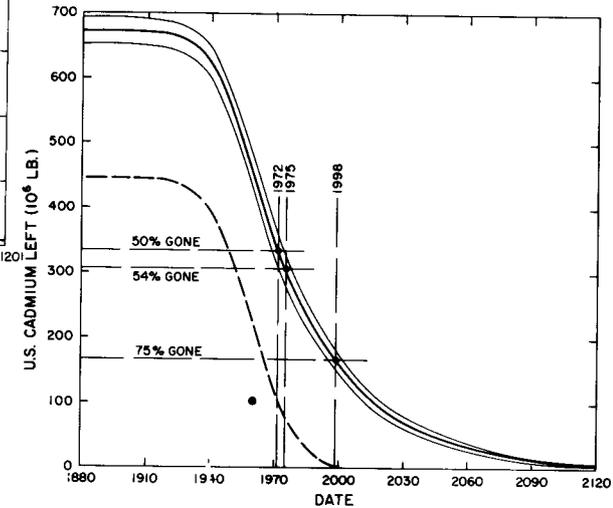
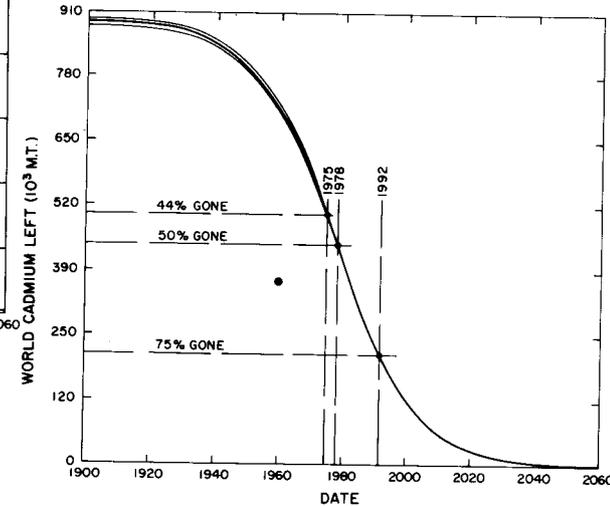
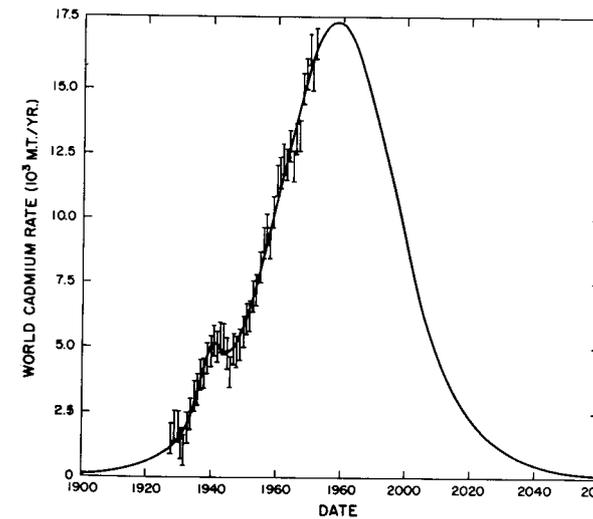


Figure 10



CHROMITE

United States: We found no production data after 1961. The sporadic production of chromite indicates that further production peaks may occur. Supportive of this possibility is the fact that the 1960 reserves estimate (●) (Frasché, 1962) is about twice as large as our Q(1960) value.

**Half date — 1955,
100% gone in 1975.**

World: The production data appear to be near peaking (~1980). As for all fits to production data that have not yet peaked, we must caution that our prediction may be pessimistic since it does not allow for any asymmetry. The 1960 reserves estimate (2.7×10^9 M.T. — not shown in the figure) (Frasché, 1962) is nine times our Q(1960) value.

**Peak date — Half date — 1981.
40% gone in 1975.**

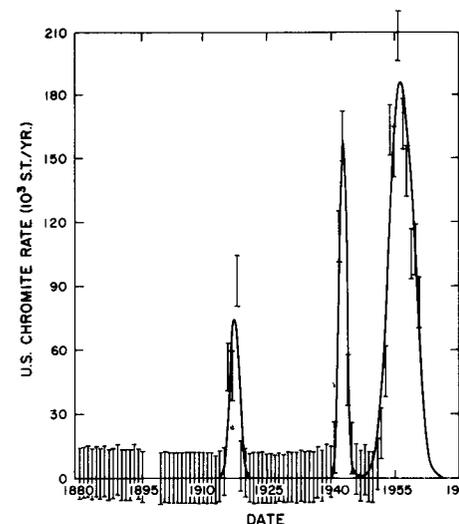


Figure 11

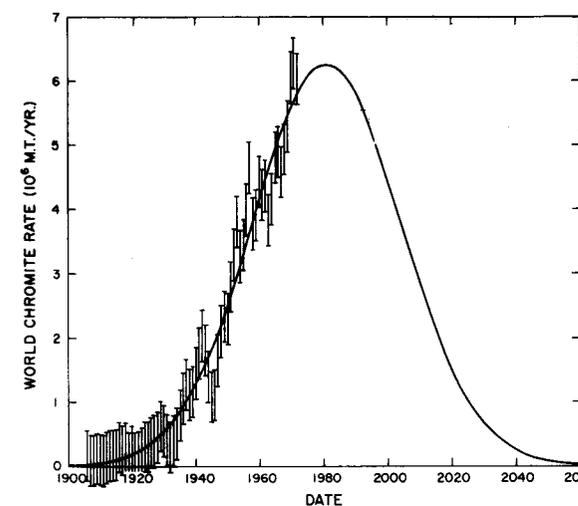
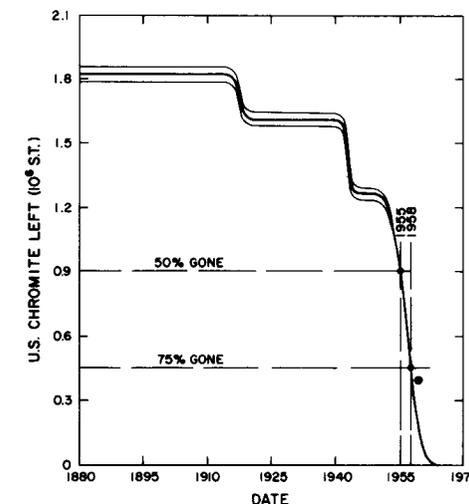
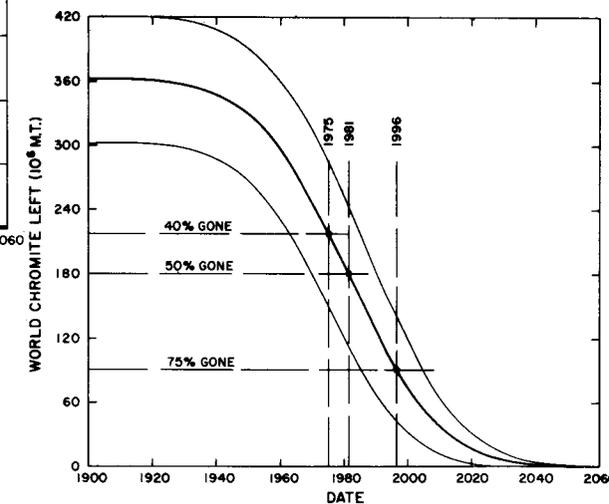


Figure 12



COBALT

United States: We found production data only for the years 1945-1959. This is not enough to make a fit.

World: We found no data for 1947-1950, 1931-1933, and prior to 1924. We predict that production will peak at ~1980. As for all fits to production data that have not yet peaked, we must caution that our prediction may be pessimistic since it does not allow for any asymmetry. Also, the 1960 reserves estimate (2×10^6 M.T. — not shown in the figure) (Frasche, 1962) is about twice our $Q(1960)$ value.

**Peak date — Half date — 1980,
41% gone in 1975.**

The available U. S. Cobalt production data are not sufficient to make a fit.

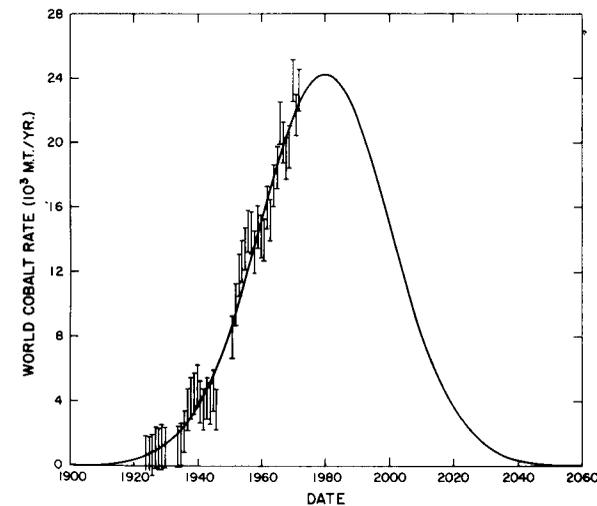
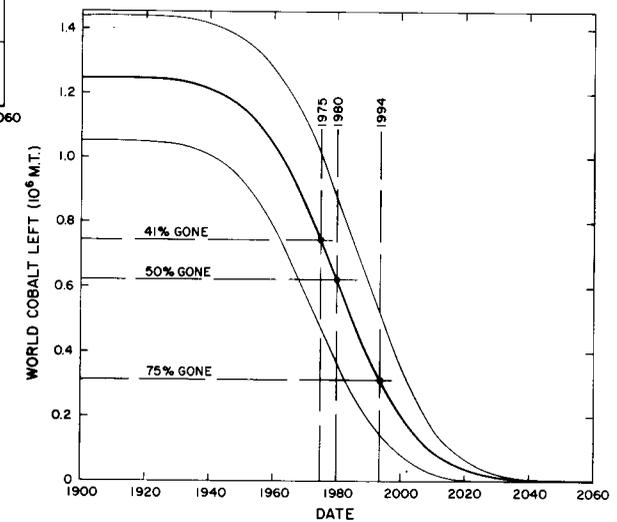


Figure 13



COPPER

United States: We use four peaks to fit the copper data because the single-peak fit has very large uncertainties. So, effectively we are assuming that the socio-technical conditions are different now for copper than they were before the mid-1900's. This is consistent with the fact that high-grade copper ores are essentially already depleted and we are mining low-grade porphyry ores now (Skinner, 1969). We predict peaking at ~2017. As for all fits to production data that have not yet peaked, we must caution that our prediction may be pessimistic since it does not allow for any asymmetry. However, the 1960 reserves estimate (●) (Frasché, 1962) is much less than our Q(1960) value.

**Peak date — 2020, Half date — 2017,
20% gone in 1975.**

World: We predict peaking at ~1988, which is one of the few cases where we predict a World peak sooner than a United States peak. As for all fits to production data that have not yet peaked, we caution that our prediction may be pessimistic since it does not allow for any asymmetry. However, the 1960 reserves estimate (●) (Frasché, 1962) is considerably lower than our Q(1960) value.

**Peak date — Half date — 1988,
34% gone in 1975.**

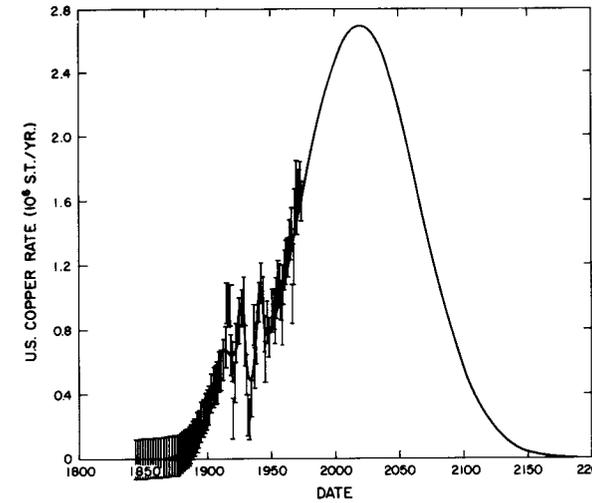


Figure 14

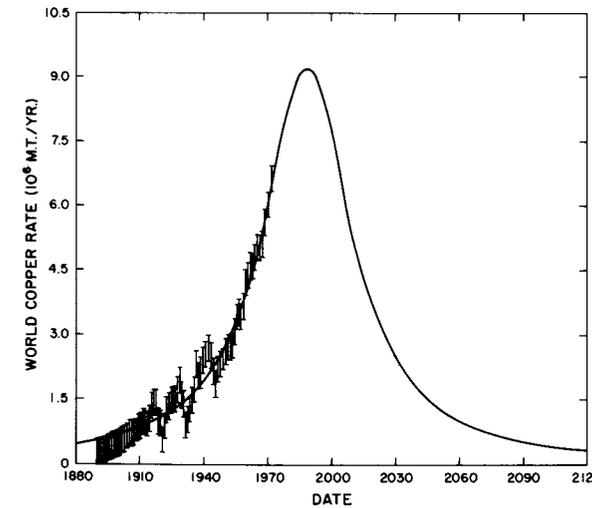
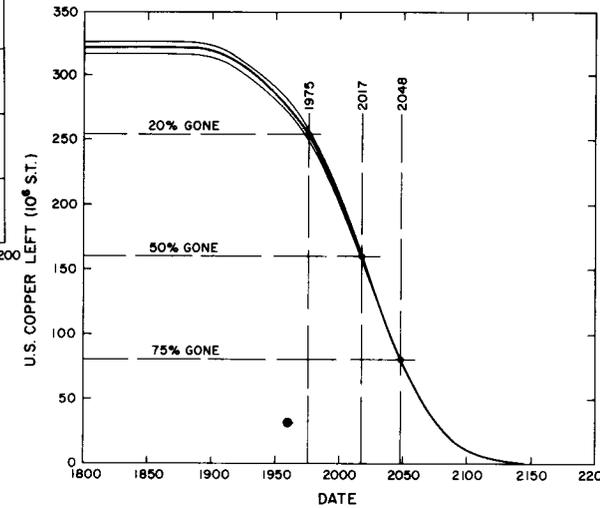
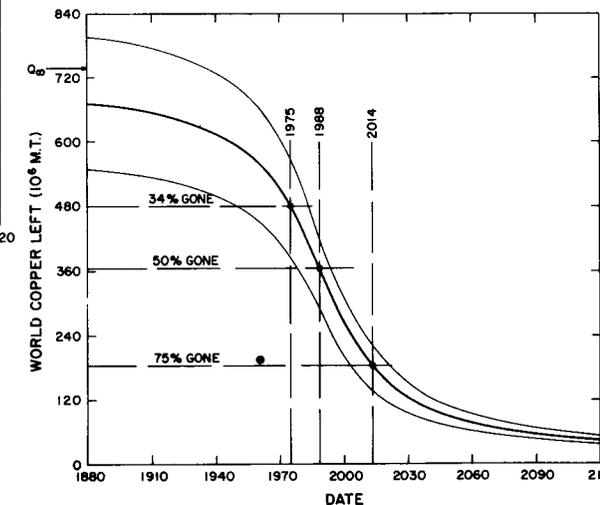


Figure 15



GOLD

United States: Data for early years were given in terms of value rather than troy ounces. We used some years where both units were given to establish the conversion ratio 0.048375 oz./\$ and used this factor to convert all data from early years to troy ounces. Despite the large transient fluctuations we were able to obtain a smooth fit for the average behavior that peaks at 1916 and has no asymmetry (solid curve). The dashed curve is a fit using a sum of four peaked functions and the dotted curve is the asymmetric fit. The three fits differ very little in $Q(t)$. The 1960 reserves estimate (●) (Frasché, 1962) is only slightly below our $Q(1960)$ value.

**Peak date — Half date — 1916,
90% gone in 1975.**

World: Data for early years were converted from money value to troy ounces as described for the United States data. The solid curve is a three-peak fit to the data and the dashed curve is a single-peak fit. We use the latter because it is more optimistic. As for all fits to production data that have not yet peaked, we must caution that our prediction may be pessimistic since it does not allow for any asymmetry. However, note that the 1960 reserves estimate (●) (Frasché, 1962) is very much less than the $Q(1960)$ value of either curve.

**Peak date — Half date — 2033,
16% gone in 1975.**

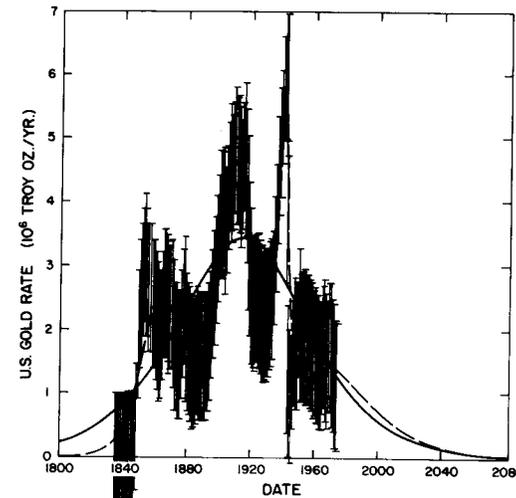


Figure 16

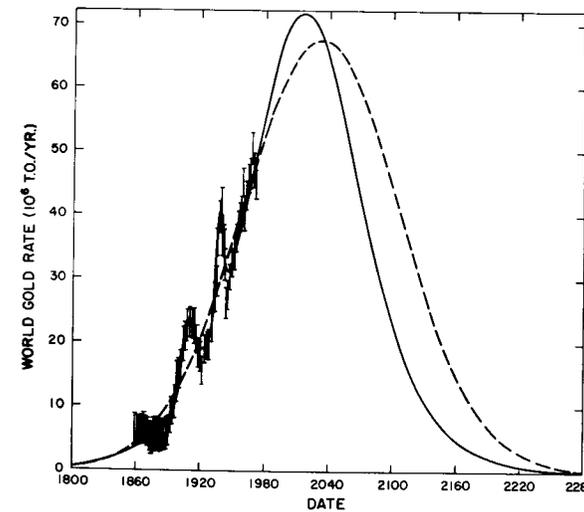
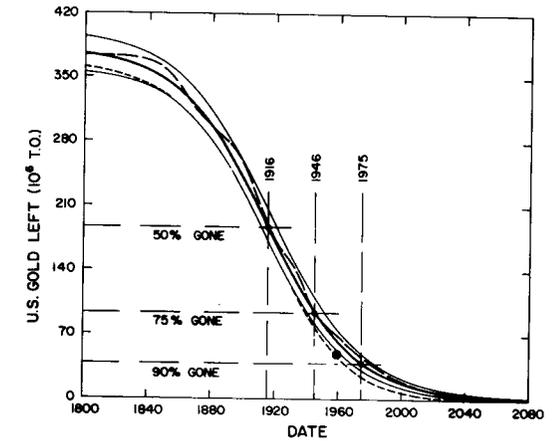
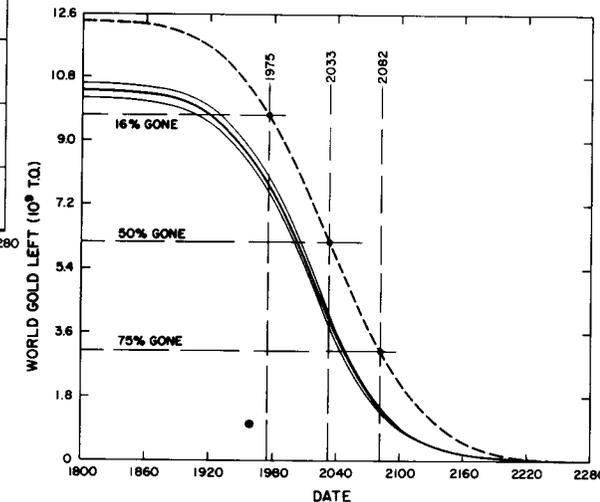


Figure 17



IRON ORE

United States: Peaking appears to have already occurred (~1962). However, there may later occur a large asymmetry which would be in agreement with the fact that the 1960 reserves estimate (●) (Frasché, 1962) is slightly above our Q(1960) value. Also, the recent peaking may be a large fluctuation like that which occurred in the 1920's.

**Peak date — Half date — 1962,
62% gone in 1975.**

World: We predict peaking at ~1992. As for all fits to production data that have not yet peaked, we must caution that our prediction may be pessimistic since it does not allow for any asymmetry. However, the 1960 reserves estimate (●) (Frasché, 1962) is slightly smaller than our Q(1960) value.

**Peak date — Half date — 1992,
26% gone in 1975.**

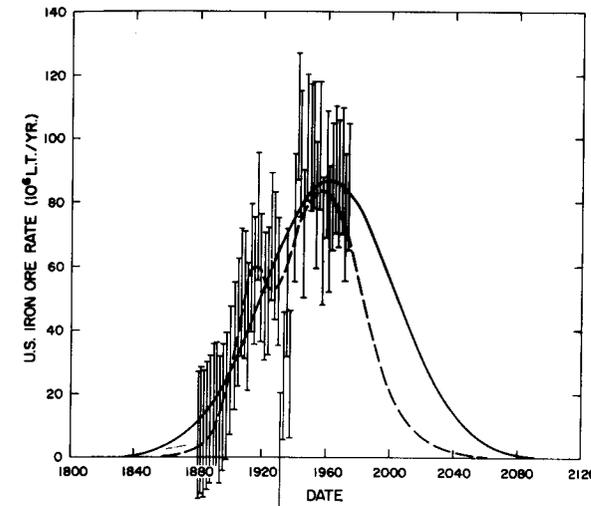


Figure 18

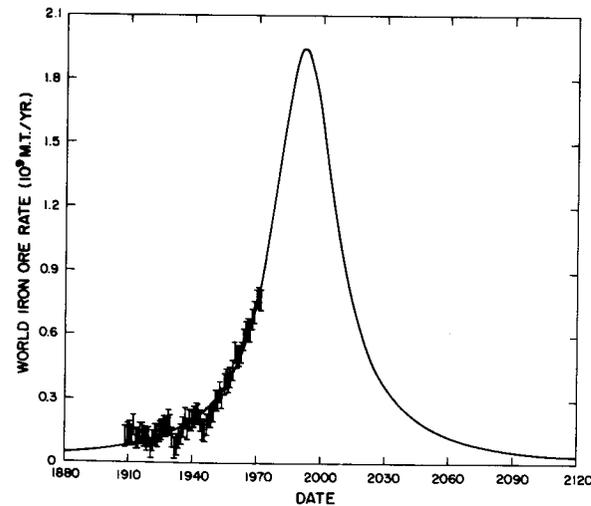
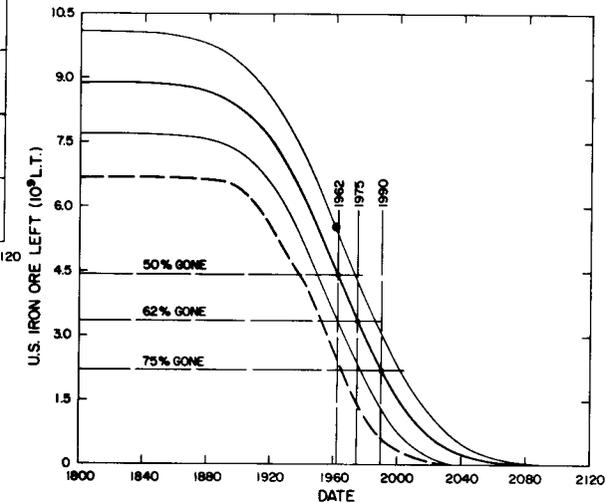
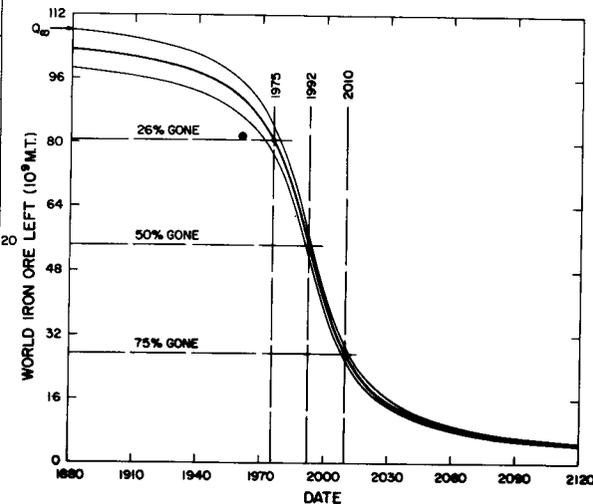


Figure 19



LEAD

United States: The best fit to the data of *Minerals Yearbook* is asymmetric (solid curve). We also show the asymmetric fit to the Department of Commerce data (dotted curve) (U.S. Dept. of Commerce, 1960). Only the *Minerals Yearbook* data are plotted. The differences between these two conflicting sets of data are insignificant for our purposes. For comparison we show a less probable symmetric fit (dashed curve) to the *Minerals Yearbook* data. We regard this prediction as rather uncertain because of the huge fluctuations. Our Q(1960) value is considerably higher than the 1960 reserves estimate (●) (Frasché, 1962) but somewhat lower than the 1968 reserves estimate (●) (Cook, 1975.).

**Peak date — 1925, Half date — 1958,
60% gone in 1975.**

World: There is a large difference between our single-peak fit (solid curves) and our four-peak fit (dashed curve) to the data. We choose the former because it is more optimistic. We predict peaking at ~2033. As for all fits to production data that have not yet peaked, we must caution that our prediction may be pessimistic since it does not allow for any asymmetry. However, both curves' Q(1960) values are much larger than the 1960 reserves estimate (●) (Frasché, 1962).

**Peak date — Half date — 2033,
16% gone in 1975.**

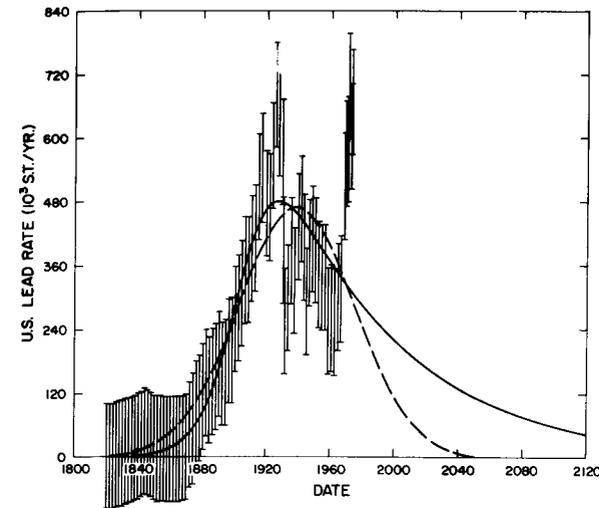


Figure 20

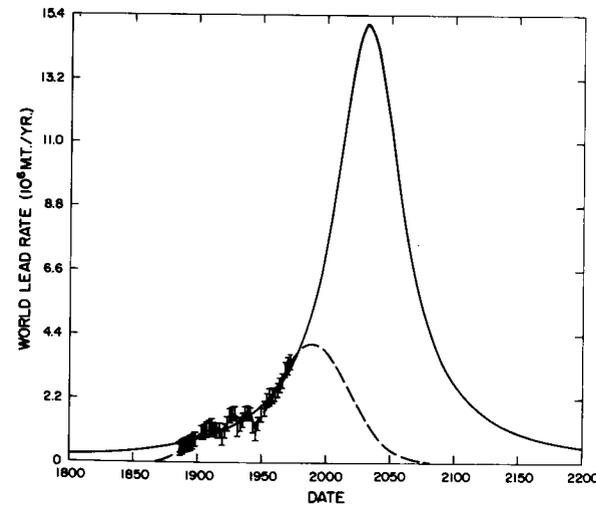
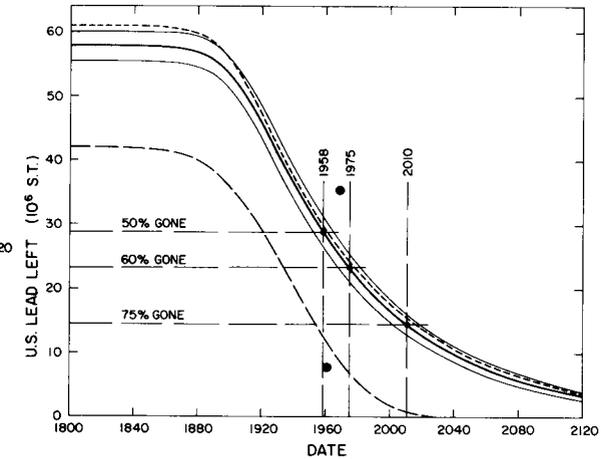
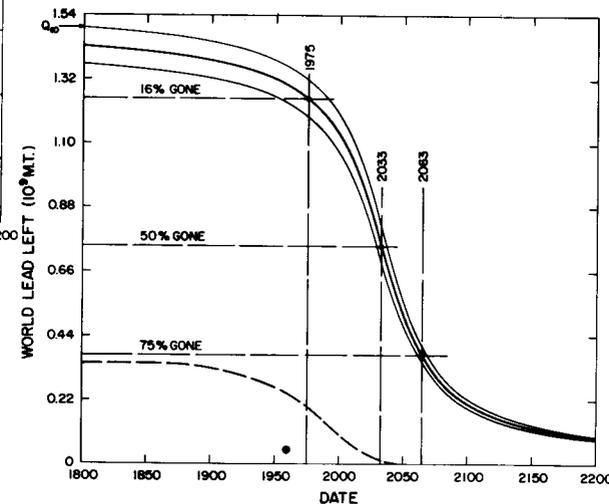


Figure 21



MAGNESIUM

United States: There is a large difference between our single-peak fit (solid curve) and our three-peak fit (dashed curve) to the data. We choose the former because it is more optimistic. It is probably ridiculous to do this magnesium fit — it probably indicates nothing more than that there is a huge amount of magnesium yet to be extracted. Already most of our magnesium metal is extracted from the huge ocean source (Park and Freeman, 1968).

**Peak date — Half date — 2004,
20% gone in 1975.**

World: There is a large difference between our single-peak fit (solid curve) and our three-peak fit (dashed curve) to the data. We choose the former because it is more optimistic. It is probably ridiculous to do this magnesium fit — it probably indicates nothing more than that there is a huge amount of magnesium yet to be extracted from the huge ocean source (Park and Freeman, 1968). According to a recent report,⁶ 60 percent of the World's magnesium is presently obtained from sea water.

**Peak date — Half date — 1995,
23% gone in 1975.**

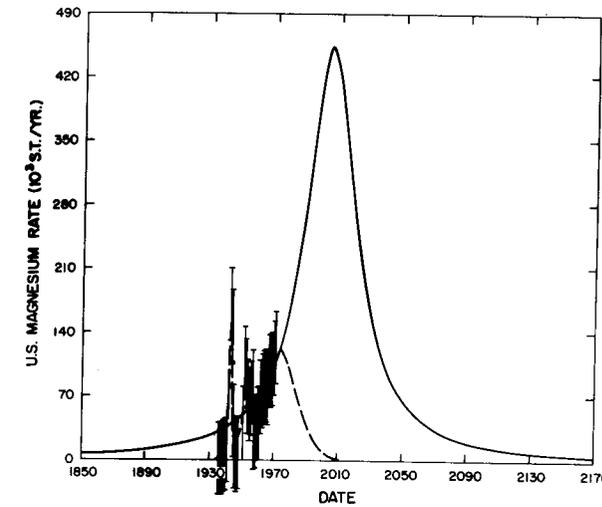


Figure 22

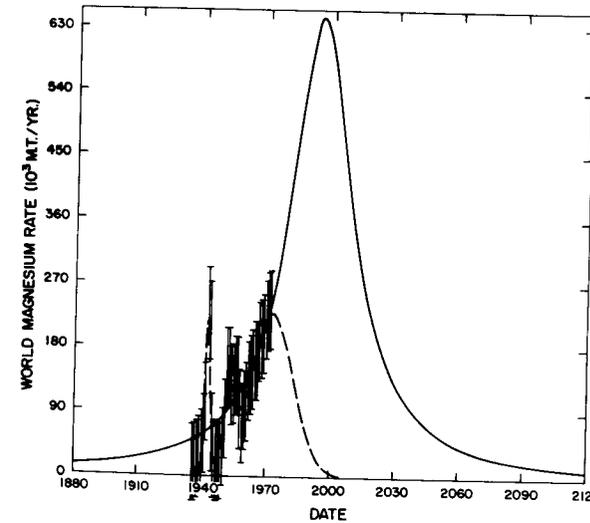
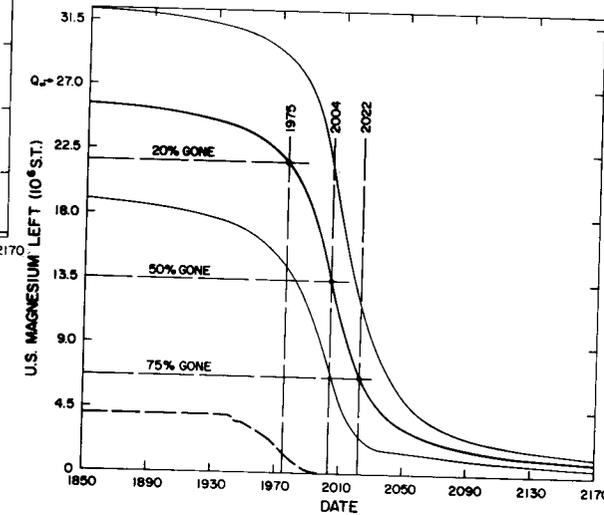
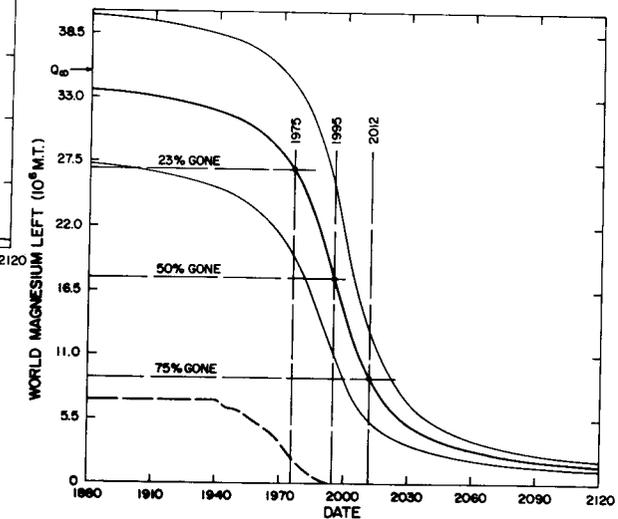


Figure 23



⁶National Advisory Council for Development Cooperation, The Hague, Netherlands, 1975.

MANGANESE ORE

United States: The huge fluctuations in manganese ore production indicate that there may be sporadic production peaks in the future, although in recent years the production rate has been essentially zero. The solid curve is a five-peak fit to the sporadic peaks, the dashed curve is a symmetric fit, and the dotted curve is a fit to the asymmetric Gompertz function. The latter is a better fit than the symmetric fit, but we use the symmetric fit for our prediction since it is more optimistic. The 1960 reserves estimate (●) (Frasché, 1962) lies between the two.

**Peak date — Half date — 1955,
90% gone in 1975.**

World: Prior to 1908 we found data only for 1900. We predict peaking soon (~1978). As for all fits to production data that have not yet peaked, we must caution that our prediction may be pessimistic since it does not allow for any asymmetry. The 1960 reserves estimate (●) (Frasché, 1962) is slightly larger than our Q(1960) value.

**Peak date — Half date — 1973,
55% gone in 1975.**

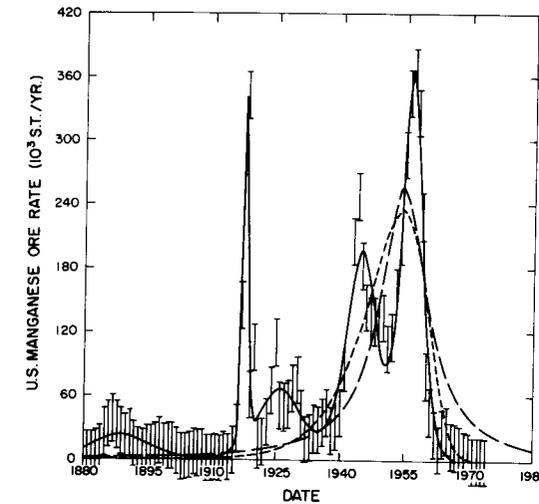


Figure 24

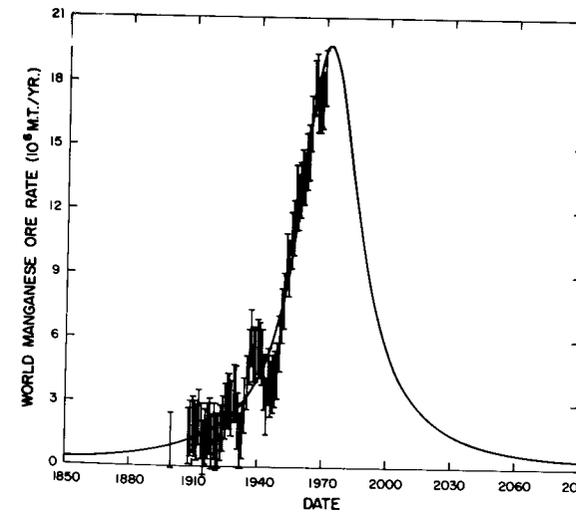
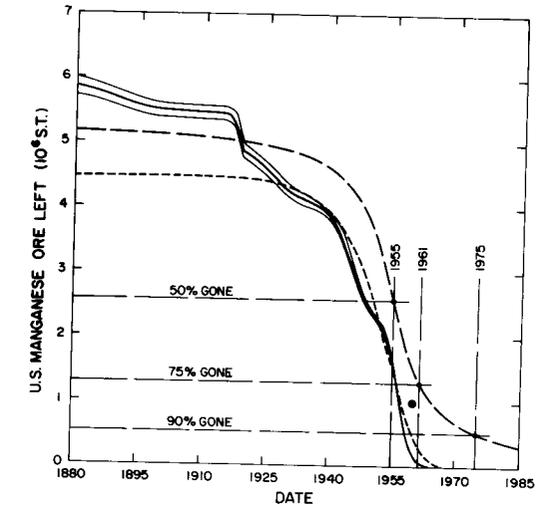
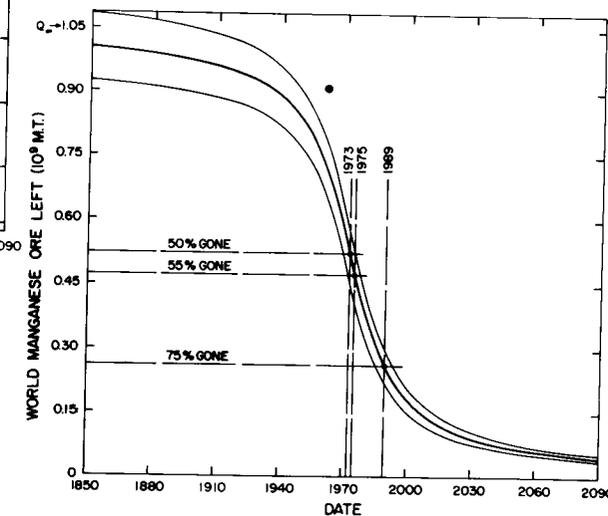


Figure 25



MERCURY

United States: Despite the many fluctuations in mercury production, there is obviously an average decreasing production rate. Mercury shares with platinum group the largest asymmetry of any mineral we have studied. Although peaking occurs at ~1870, the half date is 1916 and the three-quarter date is 1976. The 1960 reserves estimate (●) (Frasché, 1962) is about one-fourth of our $Q(1960)$ value. This case illustrates how a symmetric fit to early data could greatly underestimate future production of a mineral. One could apply the mercury asymmetry to all other minerals and confidently establish an upper limit for their future production.

**Peak date — 1870, Half date — 1916,
75% gone in 1975.**

World: The dashed curve is a four-peak fit and the solid curve is a single-peak fit. We choose the latter for our prediction because it is more optimistic. However, the 1960 reserves estimate (●) (Frasché, 1962) is in better agreement with the former fit. We regard this prediction as highly uncertain.

**Peak date — Half date — 2038,
17% gone in 1975.**

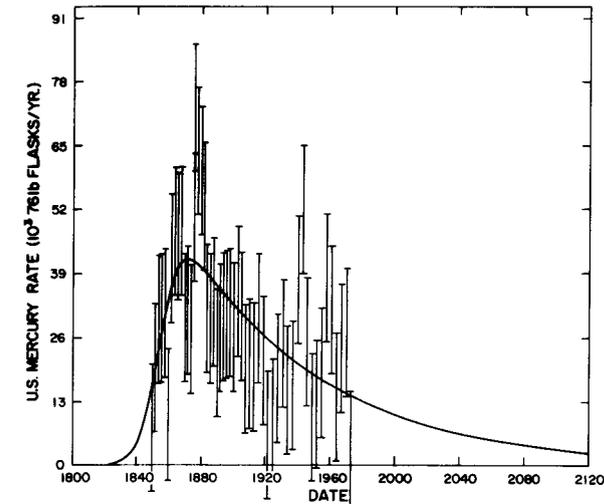


Figure 26

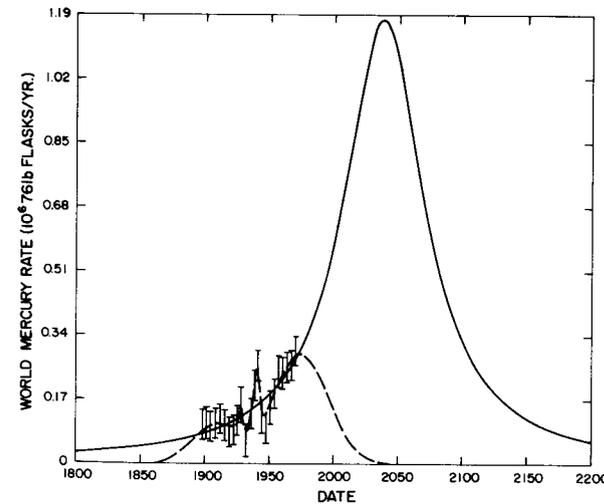
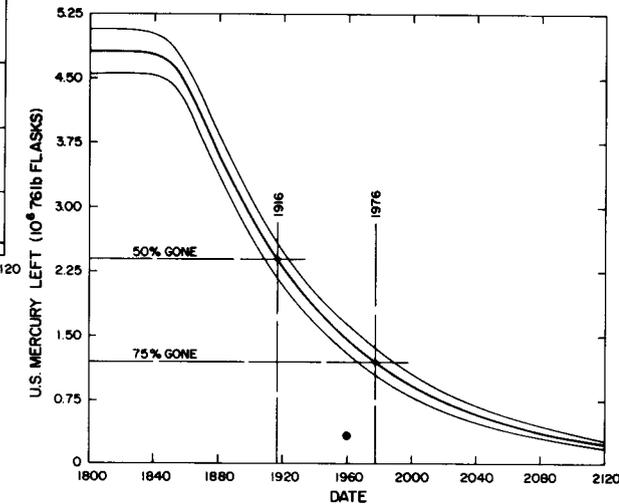
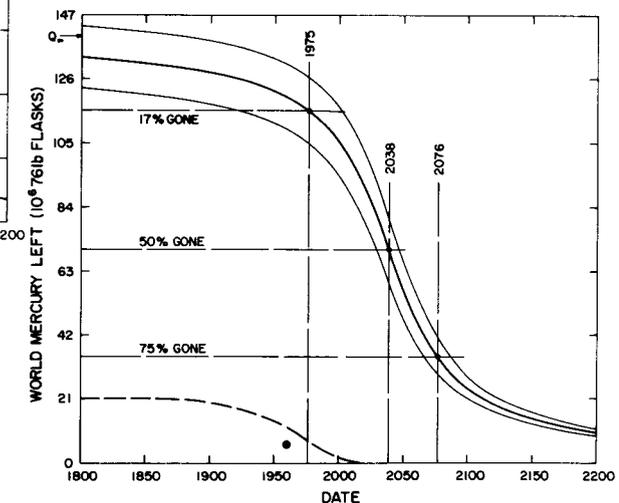


Figure 27



MOLYBDENUM

United States: We found no data prior to 1914 and for the years 1921 and 1922. We predict peaking at ~2020. As for all fits to production data that have not yet peaked, we must caution that our prediction may be pessimistic since it does not allow for any asymmetry. However, the 1960 reserves estimate (●) (Frasché, 1962) is very much less than our Q(1960) value.

**Peak date — Half date — 2020,
10% gone in 1975.**

World: We found no data prior to 1931 and for the years 1949 and 1950. We predict peaking at ~2015. As for all fits to production data that have not yet peaked, we must caution that our prediction may be pessimistic since it does not allow for any asymmetry. However, the 1960 reserves estimate (●) (Frasché, 1962) is very much less than our Q(1960) value.

**Peak date — Half date — 2014,
6% gone in 1975.**

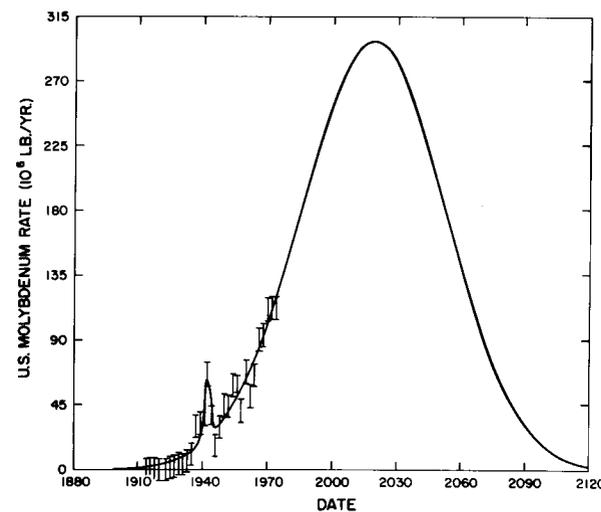


Figure 28

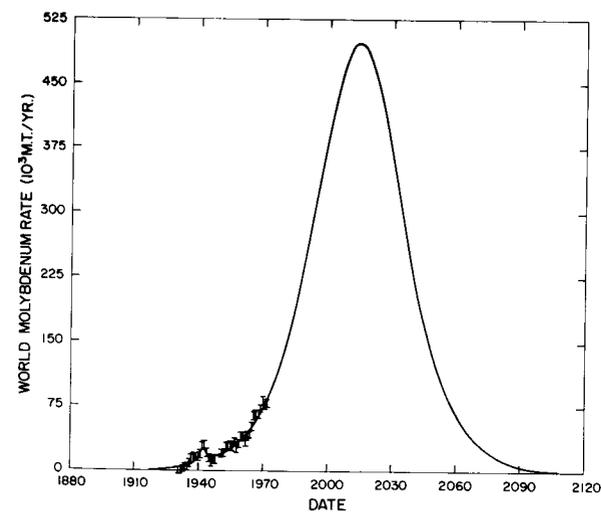
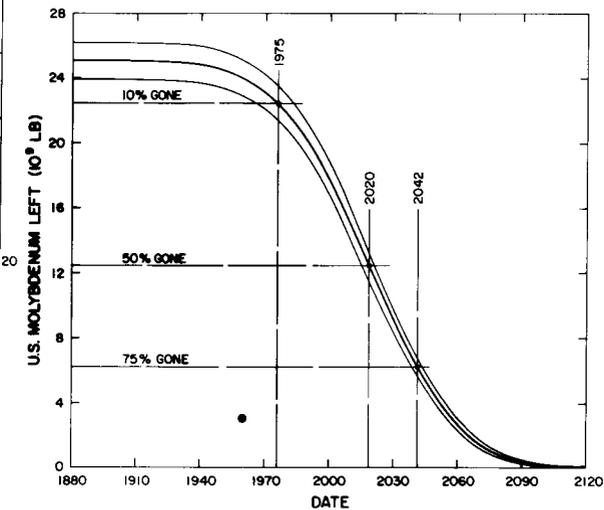
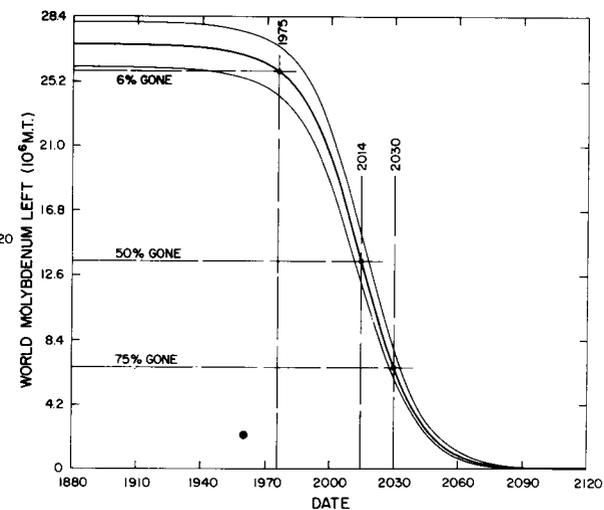


Figure 29



NICKEL

United States: We found no data for the years 1905-1910, 1912, 1973, and 1974. The solid curve is a single-peak fit and the dashed curve is a three-peak fit. We use the former for our prediction since it is more optimistic. The peak is at 1972. As for all fits to production data that are not far past the peak, we must caution that our prediction may be pessimistic since it does not allow for any asymmetry. However, the 1960 reserves estimate (●) (Frasché, 1962) is somewhat less than our Q(1960) value.

**Peak date — Half date — 1972,
58% gone in 1975.**

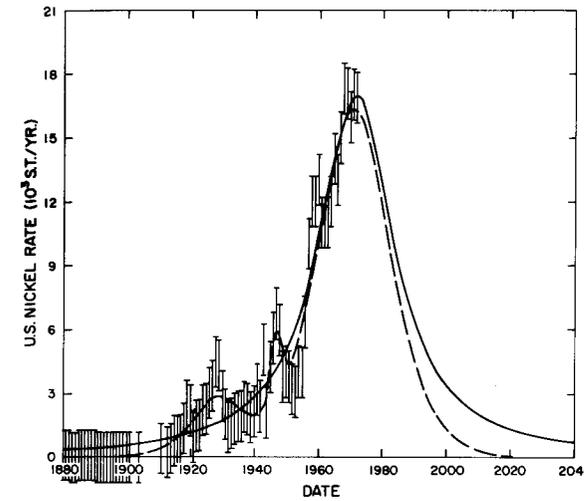


Figure 30

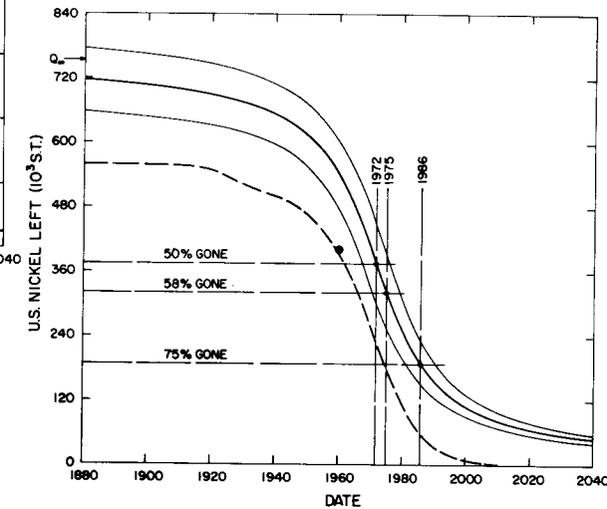


Figure 31

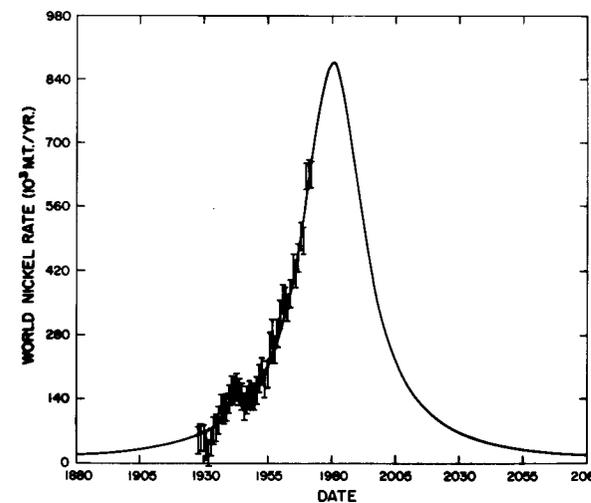
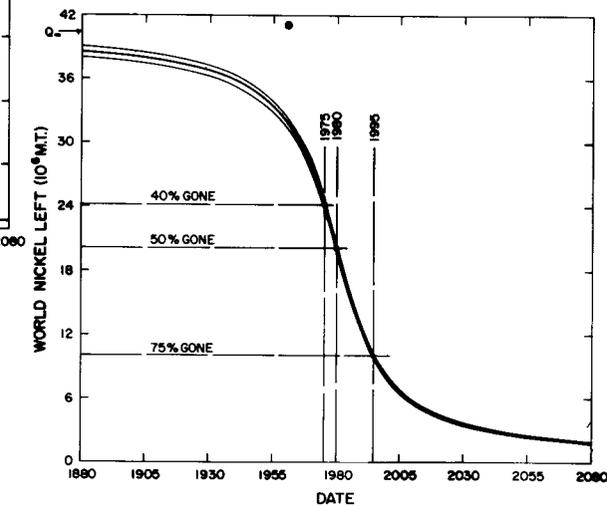


Figure 31



World: We found no data prior to 1928. We predict peaking at ~1980. As for all fits to production data that have not yet peaked, we must caution that our prediction may be pessimistic since it does not allow for any asymmetry. The 1960 reserves estimate (●) (Frasché, 1962) is somewhat above our Q(1960) value.

**Peak date — Half date — 1980,
40% gone in 1975.**

NIObIUM-TANTALUM

United States: We found no data prior to 1936, for 1940, and after 1959. The lack of data since 1959 makes any prediction suspect. If no production has occurred since ~1960, then our prediction may have some validity. However, the 1960 reserves estimate (10^8 lb — not shown in the figure) (Frasché, 1962) is one-hundred times our Q(1960) value.

**Peak date — Half date — 1957.5,
100% gone in 1975.**

World: We found no data prior to 1951. The sporadic recent production data make suspect our fit's peak at 1970. As for all fits to production data that have not yet peaked, we must caution that our prediction may be pessimistic since it does not allow for any asymmetry. Indeed, the 1960 reserves estimate (5.5×10^6 metric tons — not shown in the figure) (Frasché, 1962) is about two-hundred times our Q(1960) value.

**Peak date — Half date — 1970,
75% gone in 1975.**

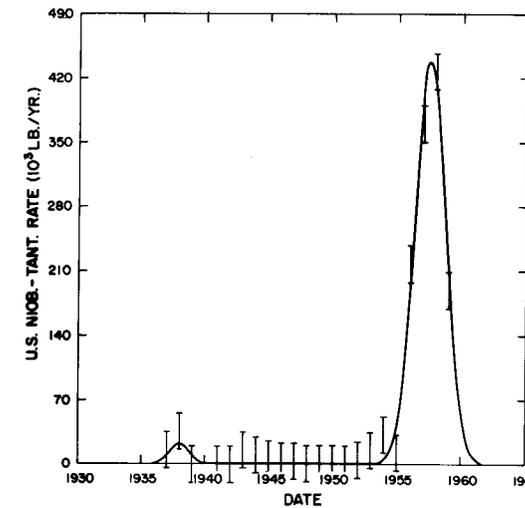


Figure 32

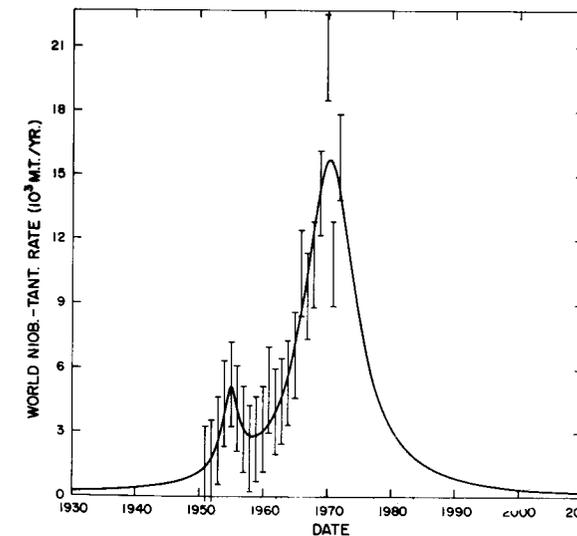
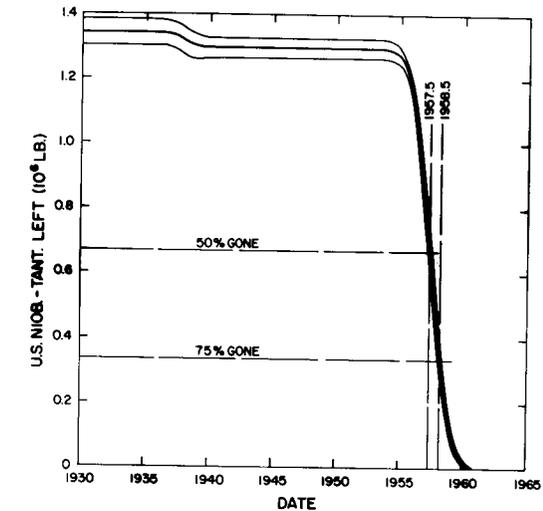
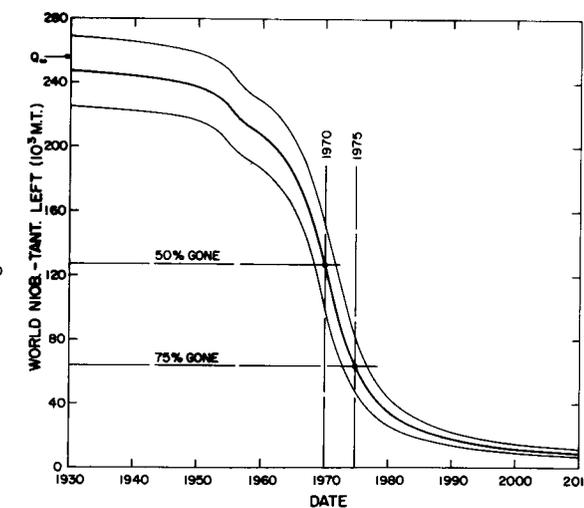


Figure 33



PLATINUM GROUP

United States: We found no data for 1972-1974. Despite the many fluctuations in platinum group production, there is obviously an average decreasing production rate. Platinum group shares with mercury the largest asymmetry of any mineral we have studied. Although peaking occurs at ~1923, the half-date is 1941 and the three-quarter date is 1964. The solid curve is the asymmetric fit and the dashed curve is a symmetric fit. The 1960 *platinum-only* reserves estimate (○) (Frasché, 1962) is about one-fourth of our Q(1960) value. This case illustrates how a symmetric fit to early data could greatly underestimate future production of a mineral. One could apply the platinum group asymmetry to all other minerals and confidently establish an upper limit for their future production.

**Peak date — 1923, Half date — 1941,
82% gone in 1975.**

World: We found no data prior to 1912. Our fit gives the peak at ~1973. As for all fits to production data that are not far past the peak, we must caution that our prediction may be pessimistic since it does not allow for any asymmetry. However, the 1960 *platinum-only* reserves estimate (○) (Frasché, 1962) is about one-fourth of our Q(1960) value.

**Peak date — Half date — 1973,
57% gone in 1975.**

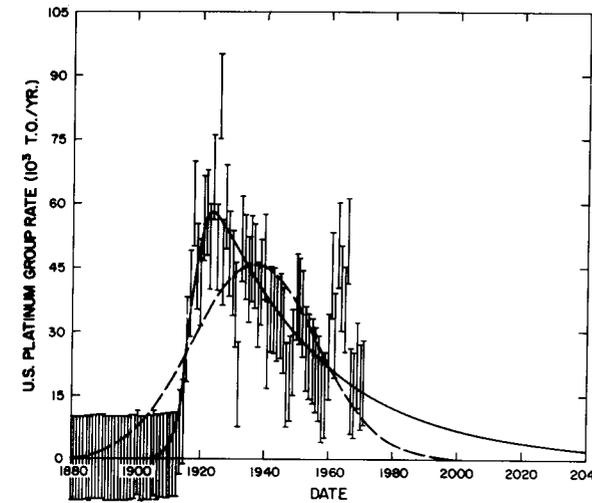


Figure 34

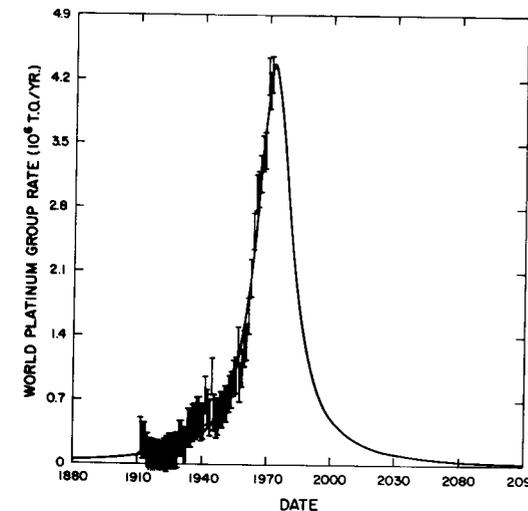
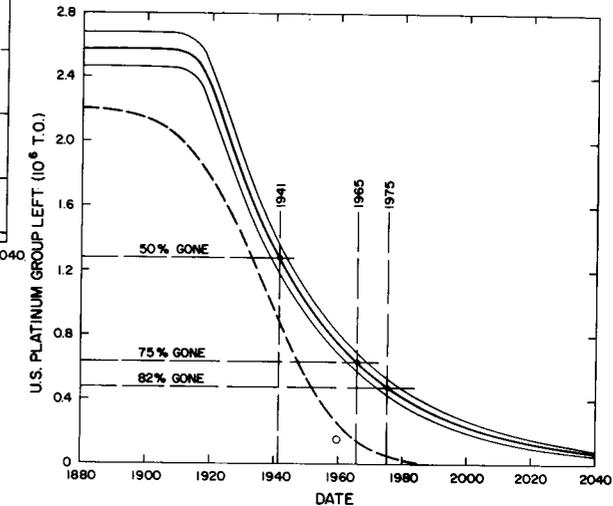
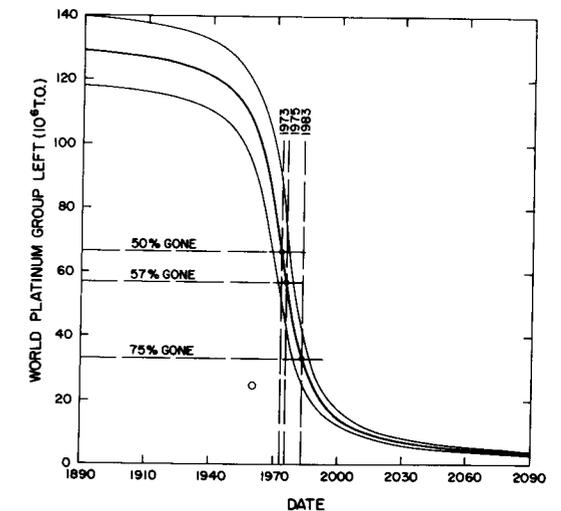


Figure 35



SELENIUM

United States: We found no data prior to 1936 and for 1949-1956. Our symmetric fit peaks at ~ 1961 . We found no evidence for asymmetry to date. However, we caution that asymmetry still could occur.

**Peak date — Half date — 1961,
79% gone in 1975.**

World: The sixteen data from 1957 to 1972 are not enough to make a prediction.

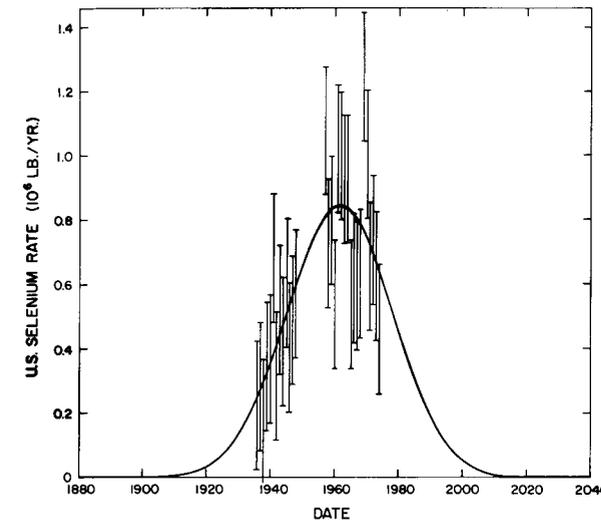
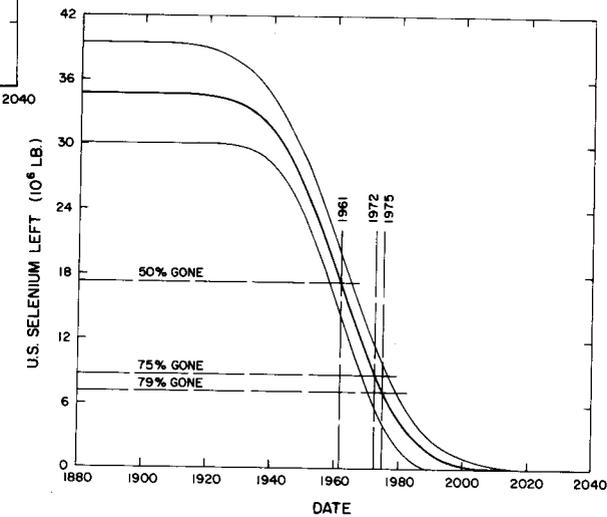


Figure 36



The available World Selenium production data are not sufficient to make a fit.

SILVER

United States: Data for early years are given in terms of value rather than troy ounces. We used some years where both units were given to establish the conversion ratio 0.77344 oz./\$ and used this factor to convert all early data to troy ounces. The dashed curve is a symmetric fit. A much better fit is achieved by allowing asymmetry (solid curve). The amount of asymmetry is about the same as for cadmium, lead, and zinc, but much less than for mercury and platinum group. The 1960 reserves estimate (●) (Frasché, 1962) is about one-third of our Q(1960) value.

**Peak date — 1908, Half date — 1938,
70% gone in 1975.**

World: The dashed curve is a symmetric fit. A slightly better fit is achieved by allowing asymmetry (solid line). The 1960 reserves estimate (●) (Frasché, 1962) is about one-fifth of our Q(1960) value.

**Peak date — 1945, Half date — 1978,
49% gone in 1975.**

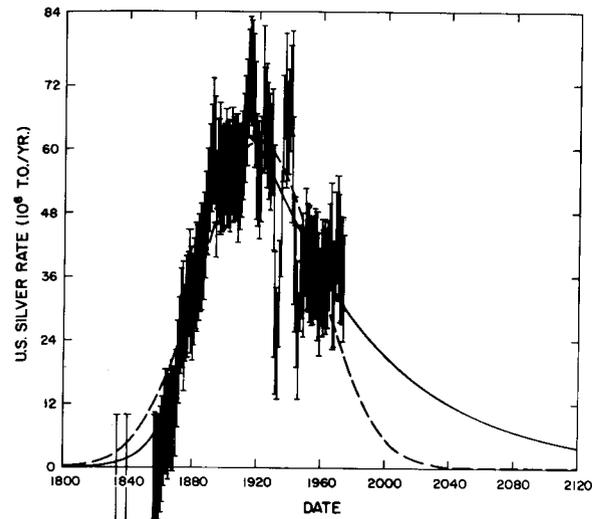


Figure 37

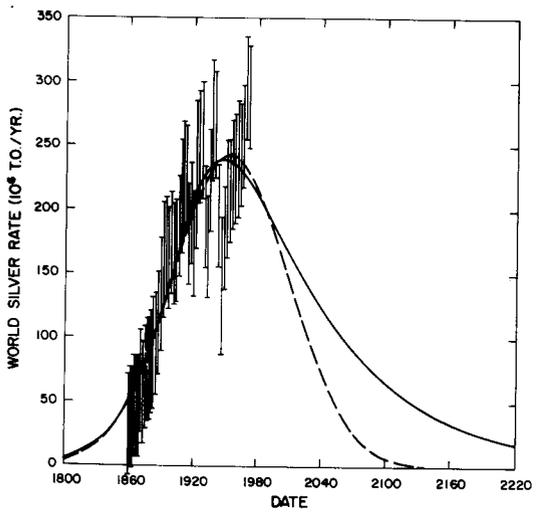
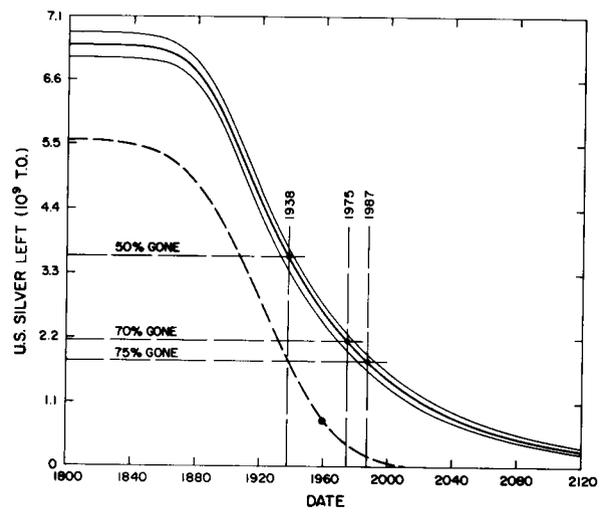
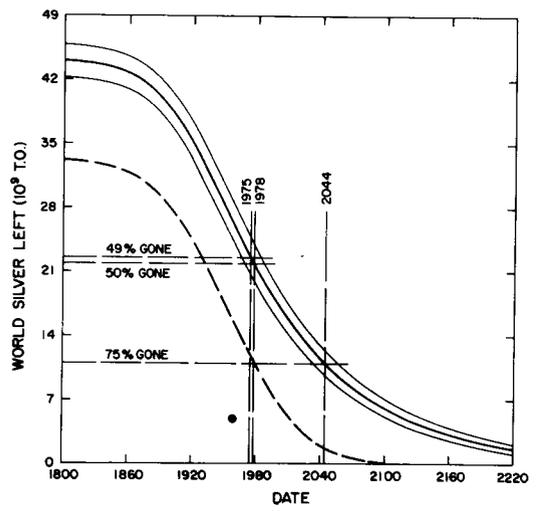


Figure 38



TELLURIUM

United States: We found no data prior to 1936 and for the years 1949-1956. The dashed curve is a two-peak fit and the solid curve is a single-peak fit. We choose the latter for our prediction because it is slightly more optimistic. We find no evidence for asymmetry to date.

**Peak date — Half date — 1964,
75% gone in 1975.**

World: The sixteen data from 1957 to 1972 are not enough to make a prediction.

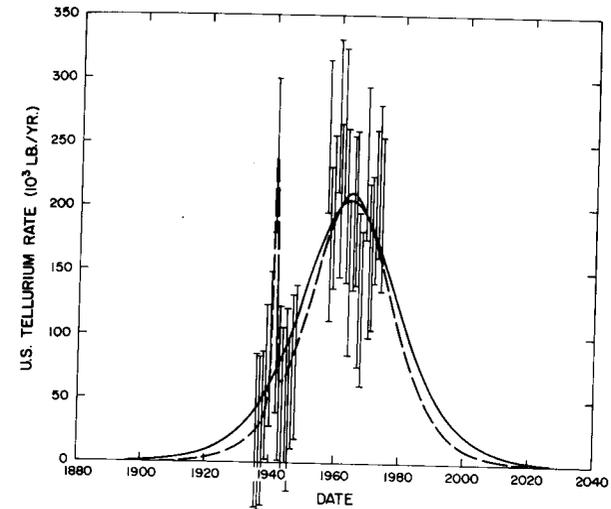
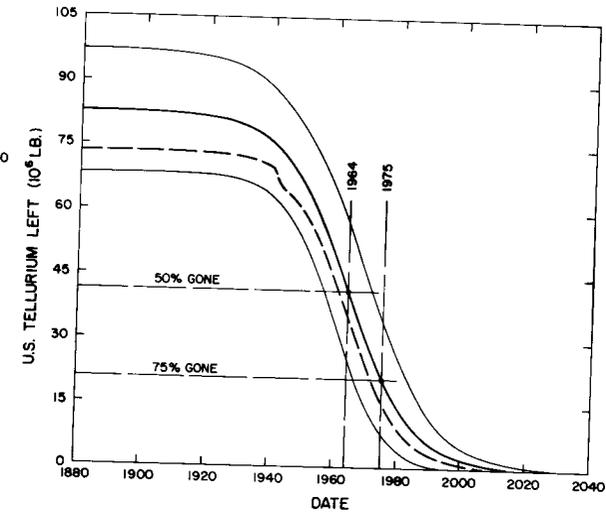


Figure 39



The available World Tellurium data are not sufficient to make a fit.

TIN

United States: We found no data for 1894-1902, 1905-1913, 1945-1946, 1956-1964, and 1967-present. There are five complete peaks on an essentially zero background. The two data for 1965 and 1966 are apparently part of another peak but are insufficient to make a prediction.

World: We found no data prior to 1921. The dashed curve is a three-peak fit and the solid curve is a single-peak fit. We use the latter because it is much more optimistic. In fact, in the light of all our predictions for other metals, it appears ridiculously optimistic (peak at ~2146). We regard this prediction as very uncertain. Also, the 1960 reserves estimate (●) (Frasché, 1962) is in better agreement with the dashed curve.

**Peak date — Half date — 2146,
11% gone in 1975.**

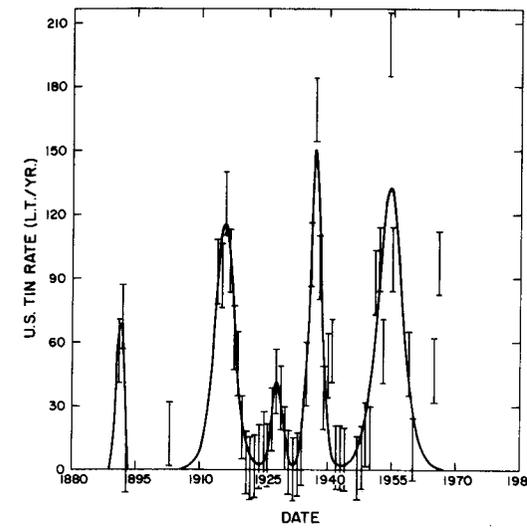


Figure 40

No prediction.

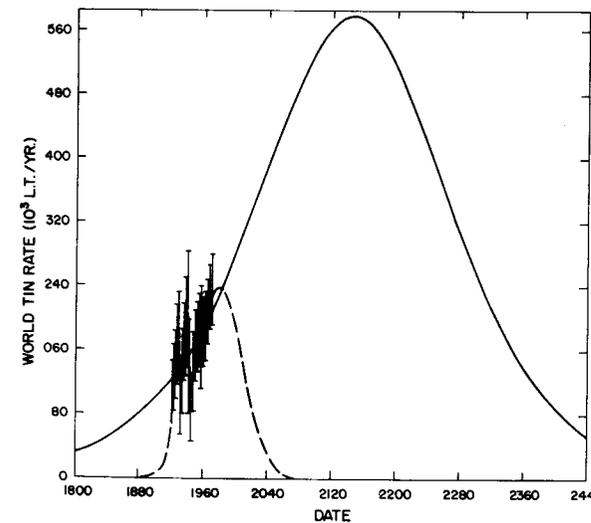
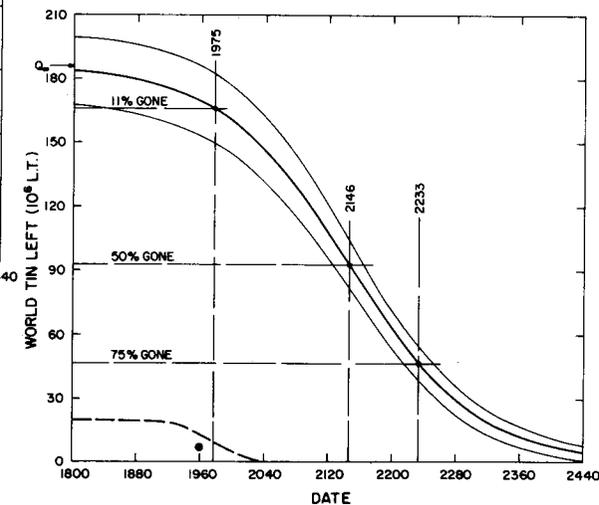


Figure 41



TITANIUM, ILMENITE

United States: We found no data prior to 1923 and for the years 1926, and 1928-1938. We find no evidence for asymmetry to date. However, since it is not far past peaking we must caution that an asymmetry could occur. Also, this peak could be a short-term fluctuation.

**Peak date — Half date — 1966,
75% gone in 1975.**

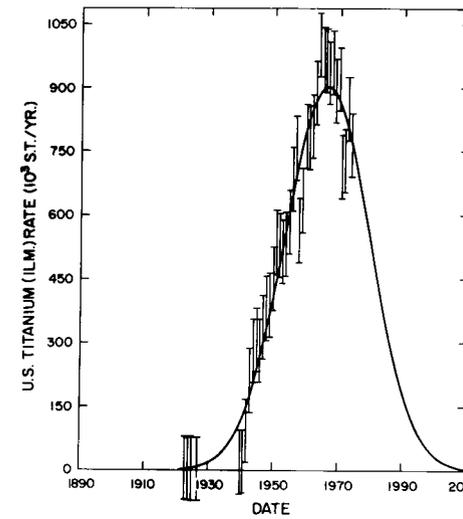


Figure 42

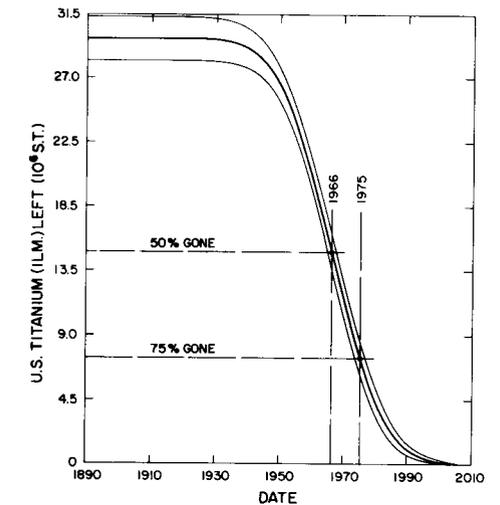


Figure 43

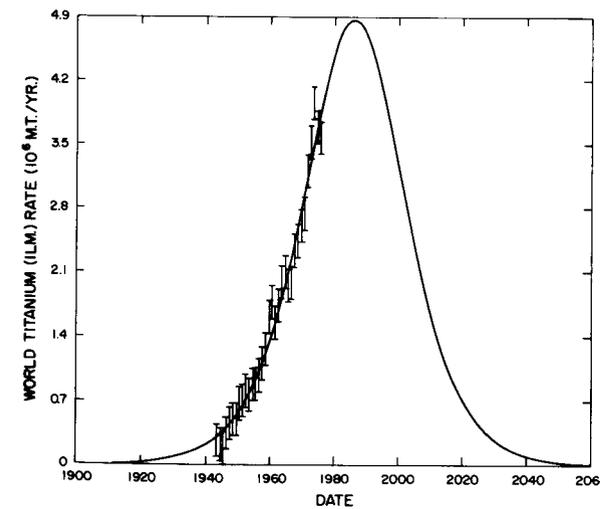
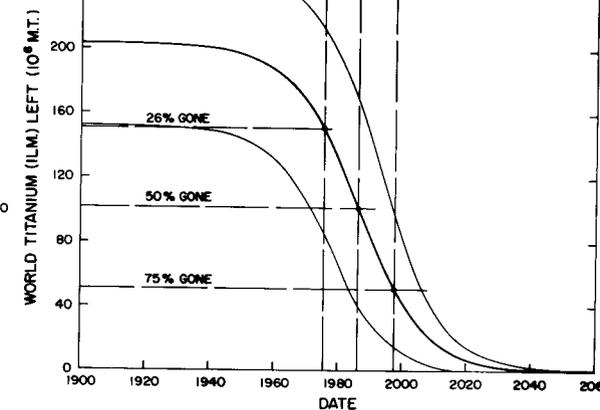
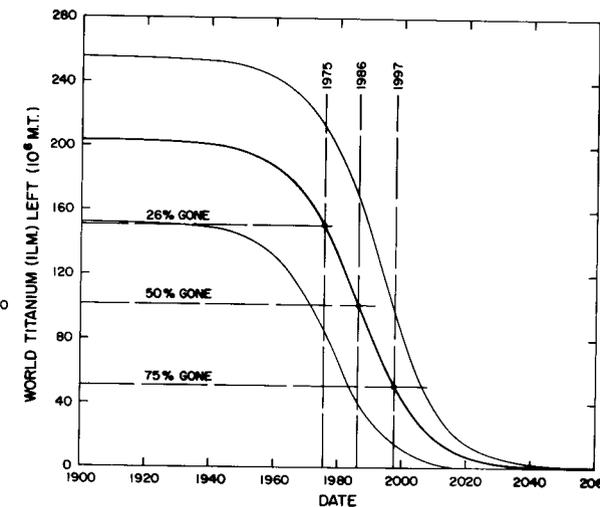
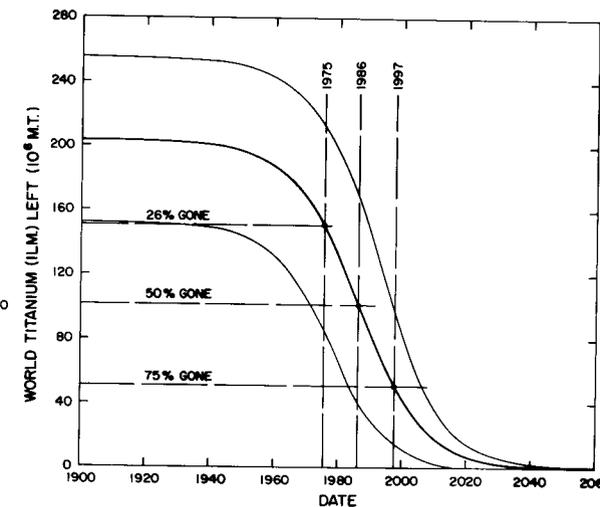


Figure 43



World: We found no data prior to 1940. We predict peaking at ~1985. As for all fits to production data that have not yet peaked, we caution that our prediction may be pessimistic since it does not allow for any asymmetry.

**Peak date — Half date — 1986,
26% gone in 1975.**

TITANIUM, RUTILE

United States: We found no data for the years 1893, 1902-1903, 1905-1909, 1911, 1921, 1926-1939, 1950-1952, and 1965-present. The missing data for the last ten years are necessary to determine if there is an asymmetry. The peak could be a short-term fluctuation.

**Peak date — Half date — 1958,
90% gone in 1975.**

World: We found no data prior to 1940. We show peaking in 1970. As for all fits to production data that are not far past peaking, we must caution that our prediction may be pessimistic since it does not allow for any asymmetry.

**Peak date — Half date — 1970,
70% gone in 1975.**

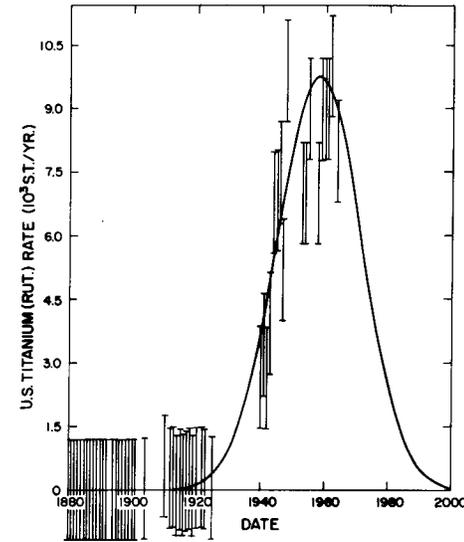


Figure 44

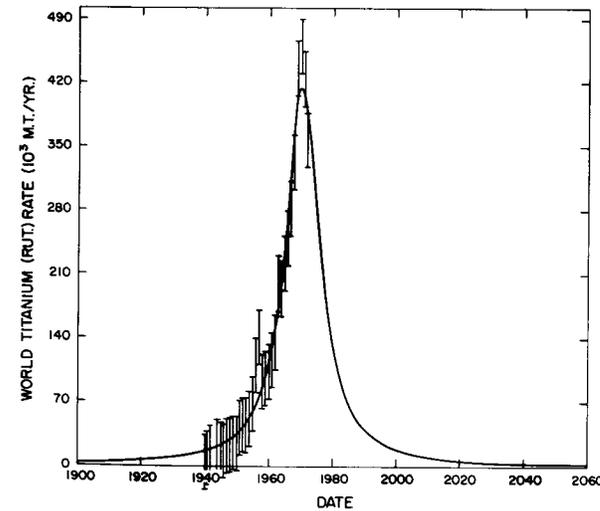
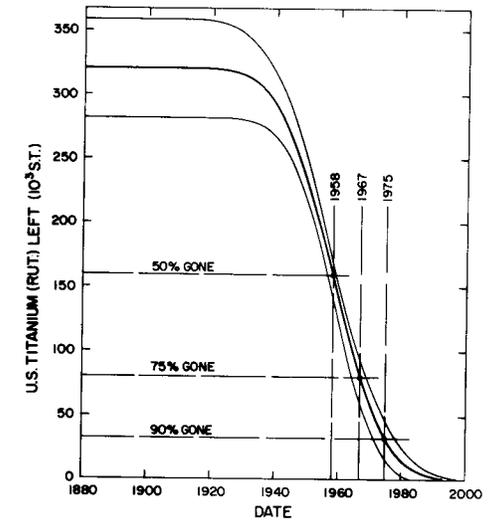
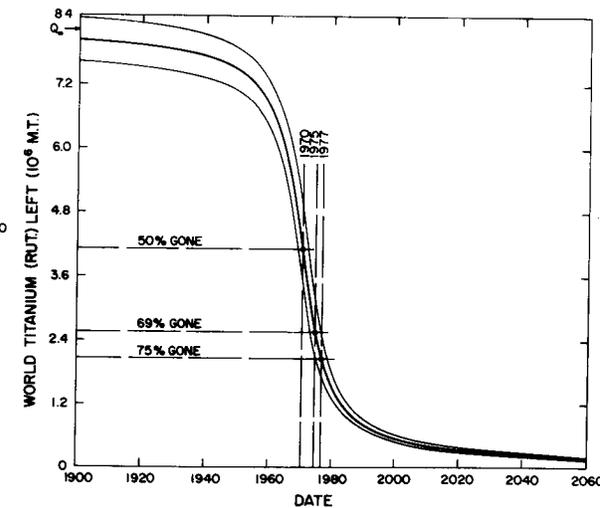


Figure 45



TUNGSTEN ORE

United States: We found no data prior to 1900 and for the years 1921-1922. For the years 1966-1971 the data are reported in terms of tungsten content rather than ore mass on a 60% WO_3 basis. We used molecular weights to calculate a ratio of 0.4758 for tungsten content over ore mass on a 60% WO_3 basis, and used this ratio to calculate ore mass for the tungsten-content data. The dashed curve is a four-peak fit and the solid curve is a single-peak fit. We use the latter because it is more optimistic. It is too soon after peaking to observe any asymmetry. Our Q(1960) value is larger than the 1960 reserves estimate (●) (Frasché, 1962).

**Peak date — Half date — 1962,
72% gone in 1975.**

World: We found no data prior to 1905 and for 1949. The data after 1962 are reported in tungsten content. We used the ratio described for the United States data to convert these data to ore mass on a 60% WO_3 basis. We get peaking in 1971. As for fits to all production data that are not far past peaking, we must caution that our prediction may be pessimistic since we do not allow for any asymmetry. However, the 1960 reserves estimate (●) (Frasché, 1962) is slightly below our Q(1960) value.

**Peak date — Half date — 1971,
57% gone in 1975.**

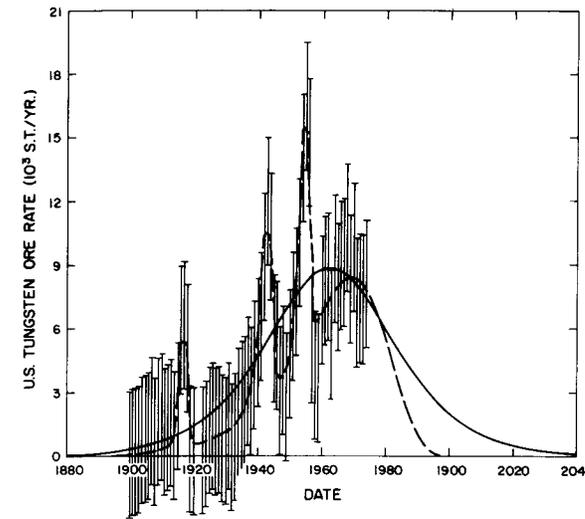


Figure 46

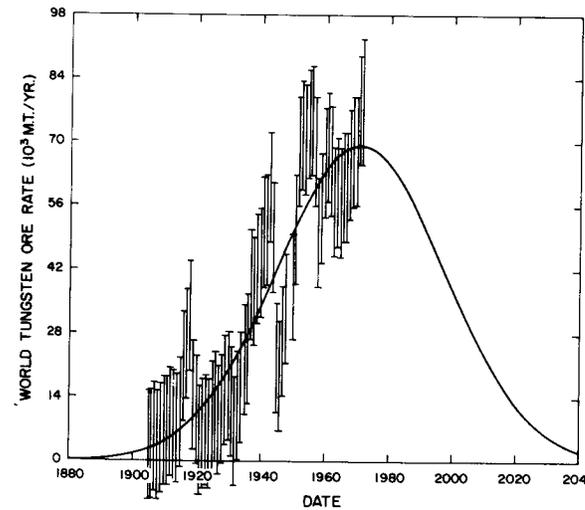
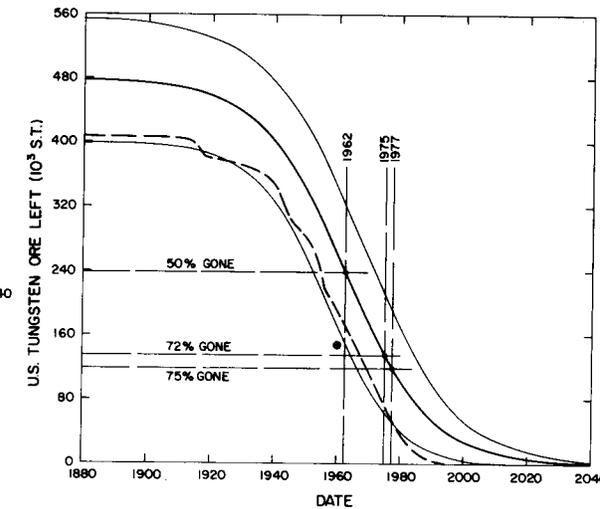
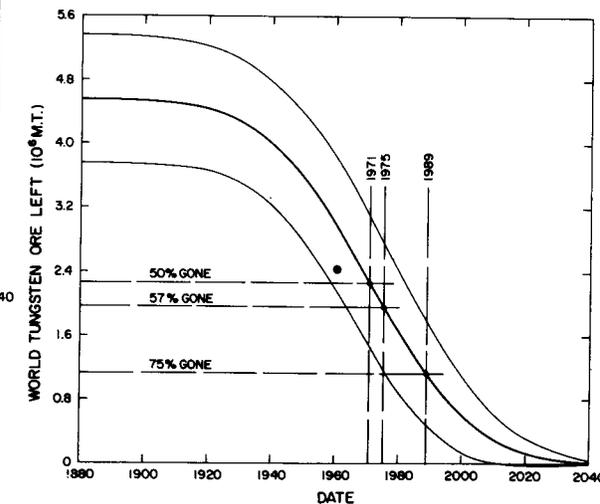


Figure 47



URANIUM OXIDE

United States: We found no data prior to 1958. (There are combined uranium plus vanadium data for some years.) The seventeen available data are not enough to make a prediction.

World: We found no data prior to 1958. The seventeen available data are not enough to make a prediction.

**NO FIT
INSUFFICIENT DATA**

VANADIUM

United States: We found no data prior to 1940 and for 1973-1974. Our fit peaks in 1968. As for all fits to production data that are not far past peaking, we must caution that our prediction may be pessimistic since we do not allow for any asymmetry. Also, the 1960 reserves estimate (6×10^5 S.T. — not shown in figure) (Frasché, 1962) is about five times our $Q(1960)$ value.

**Peak date — Half date — 1968,
69% gone in 1975.**

World: We found no data prior to 1933 and for 1934-1935. We predict peaking at ~1983. As for all fits to production data that have not yet peaked, we must caution that our prediction may be pessimistic since we do not allow for any asymmetry. However, the 1960 reserves estimate (●) (Frasché, 1962) is about one-half of our $Q(1960)$ value.

**Peak date — Half date — 1983,
18% gone in 1975.**

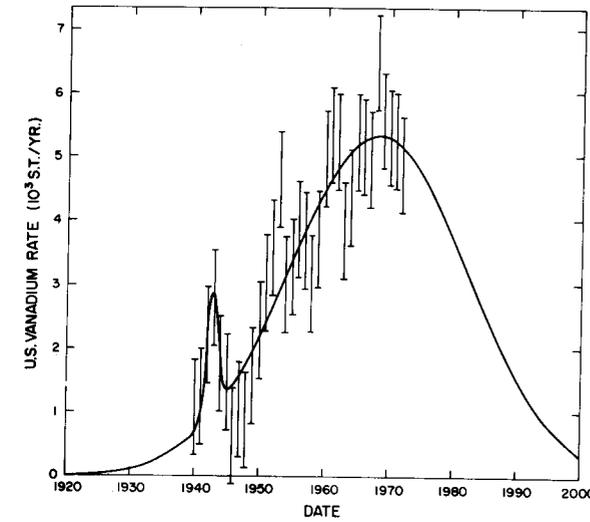


Figure 48

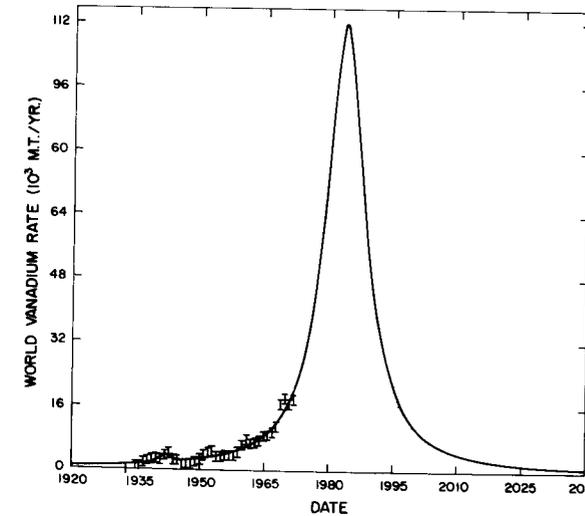
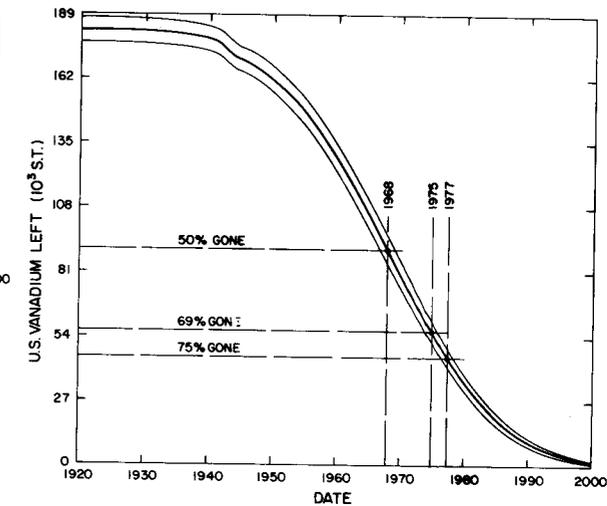
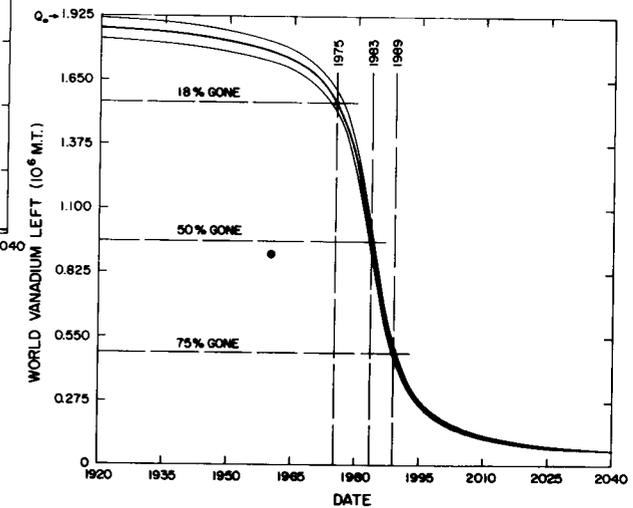


Figure 49



ZINC

United States: The best fit to the data of *Minerals Yearbook* is asymmetric (solid curve). We also show the asymmetric fit to the Department of Commerce Data (dotted curve) (U. S. Department of Commerce, 1960, 1965). Only the *Minerals Yearbook* data are plotted. The differences between these two conflicting sets of data are insignificant for our purposes. For comparison we show a slightly less probable symmetric fit (dashed curve) to the *Minerals Yearbook* data. Since the production data are not far past peaking, our asymmetric prediction has a large uncertainty. However, our Q(1960) value is slightly higher than the 1960 reserves estimate (●) (Frasché, 1962) and is in good agreement with the 1968 reserves estimate (●) (Cook, 1975).

Peak date — 1943, Half date — 1968,

54% gone in 1975.

World: We predict peaking at ~1994. As for all production data that have not yet peaked, we must caution that our prediction may be pessimistic since it does not allow for any asymmetry. However, the 1960 reserves estimate (●) (Frasché, 1962) is about ten times smaller than our Q(1960) value.

Peak date — Half date — 1994,

27% gone in 1975.

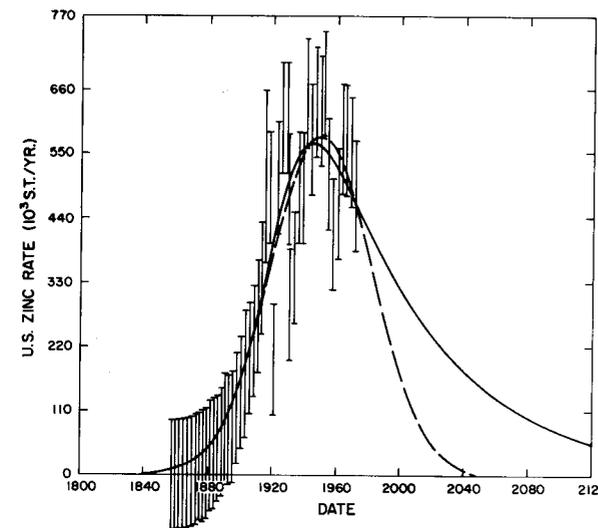


Figure 50

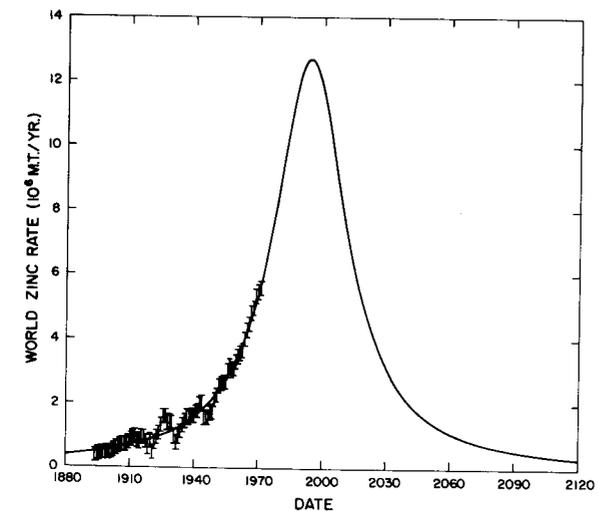
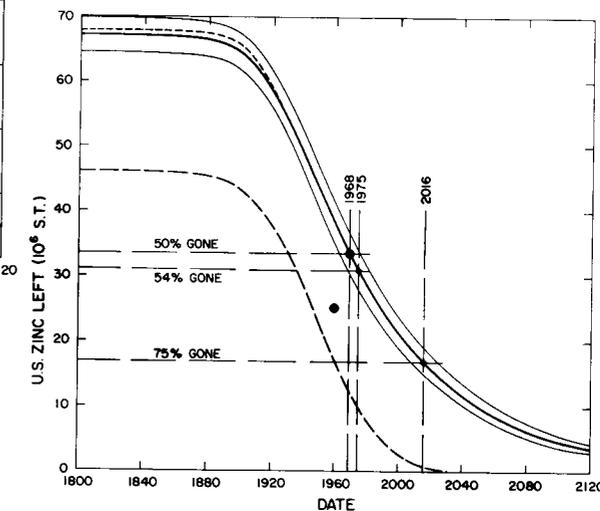
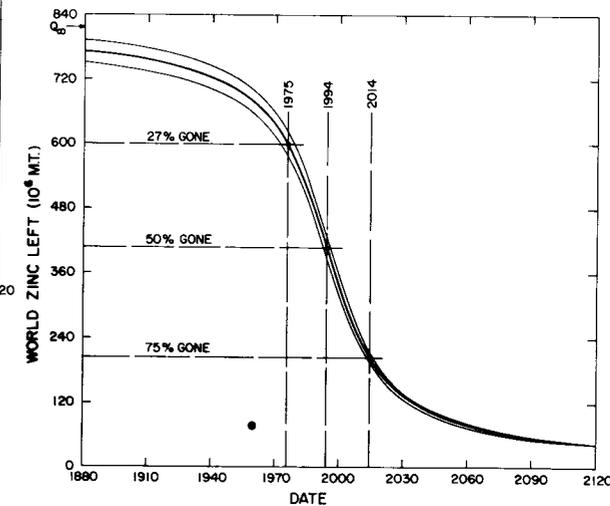


Figure 51



II. MINERAL FUELS

CRUDE OIL

United States: The data collected in Elliott, et al. (Elliott, 1968) differ somewhat from the data in *Minerals Yearbook*, but not enough to matter for our purposes. We use *Minerals Yearbook* data. The solid curve is a fit to the data through 1974, the dashed curve is a fit to the data through 1963, and the dotted curve is a fit to the data through 1953. The three predictions do not differ very much. Thus, our prediction could have been made in 1953 or perhaps earlier. As for all fits to production data that have not yet peaked we must caution that our prediction may be pessimistic since we have not allowed for any asymmetry. The 1975 reserves estimate (●) (NAS-NRC, 1975) is almost equal to our Q(1975) value.

**Peak date — Half date — 1984,
40% gone in 1975.**

World: Our best fit (solid curve) yields an unexpectedly high value for the peak date: ~2034. The dashed curve is a close-up of the fit in the region where the data exist. The 1975 reserves estimate (●) (NAS-NRC, 1975) is about one-tenth of our Q(1975) value.

**Peak date — Half date — 2034,
1.5% gone in 1975.**

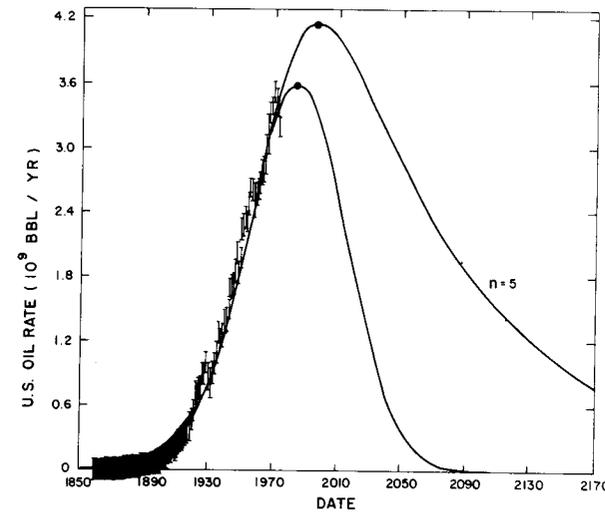


Figure 52

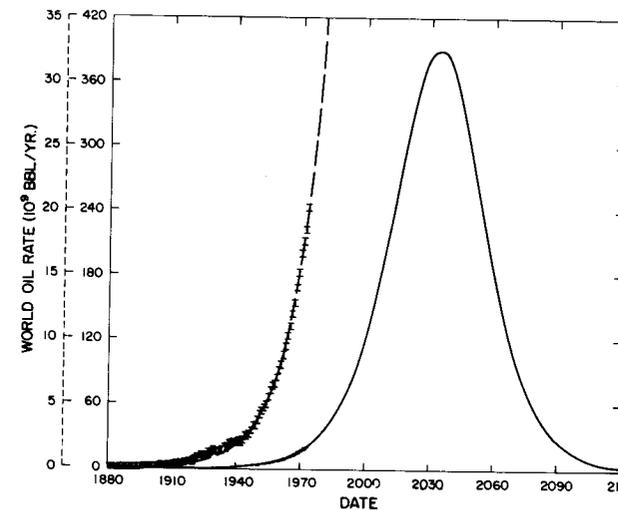
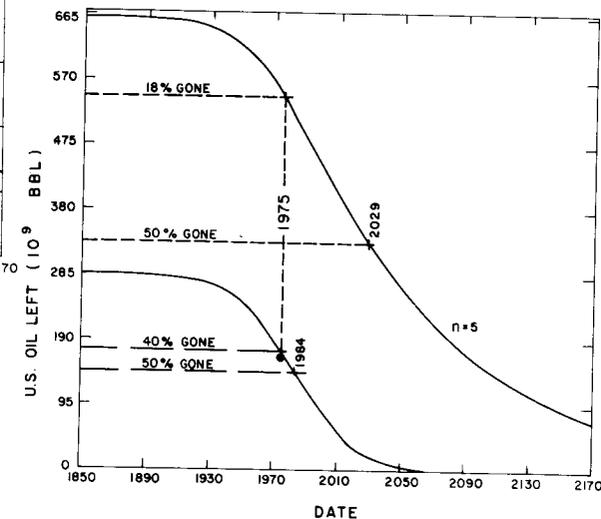
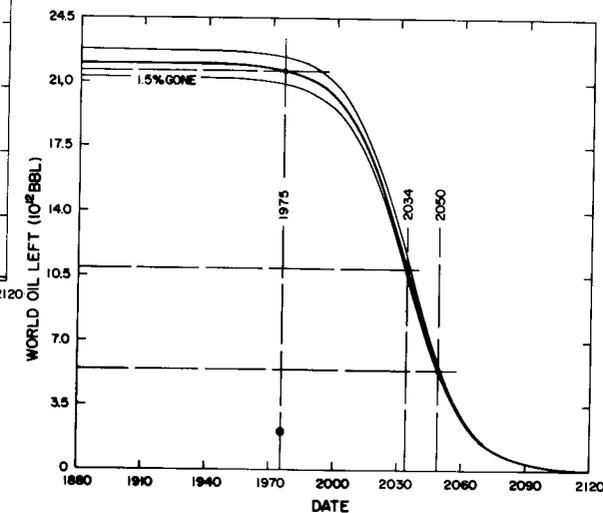


Figure 53



NATURAL GAS

United States: We found no data prior to 1900. The data collected in Elliott, et al. (Elliott, 1968) differ somewhat from the data in *Minerals Yearbook*, but not enough to matter for our purposes. We use the *Minerals Yearbook* data. We predict peaking in 1981. As for all fits to production data that have not yet peaked, we must caution that our prediction may be pessimistic since it does not allow for any asymmetry. The 1975 reserves estimate (●) (NAS-NRC, 1975) is slightly larger than our Q(1975) value.

**Peak date — Half date — 1981,
38% gone in 1975.**

World: We found no data in *Minerals Yearbook* prior to 1947. We used data from the United Nations *Statistical Yearbook* (U. N., 1974) for the years 1937-1946. We predict peaking at ~1998, but the uncertainty is large. The 1975 reserves estimate ($8 \times 10^{15} \text{ ft}^3$ — not shown in figure) (NAS-NRC, 1975) is about 50 percent larger than our Q(1975) value.

**Peak date — Half date — 1998,
11% gone in 1975.**

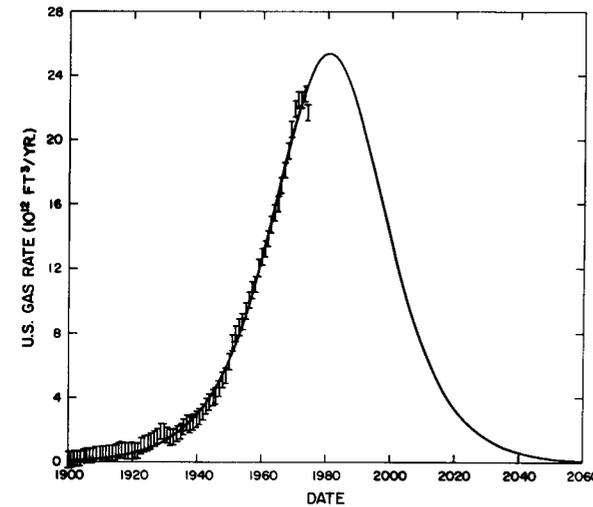


Figure 54

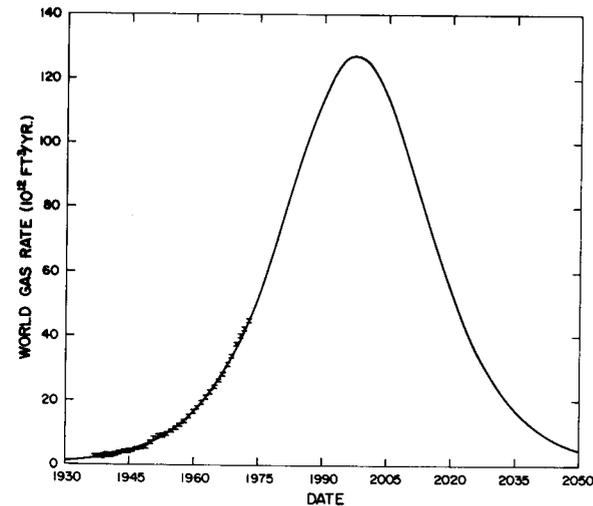
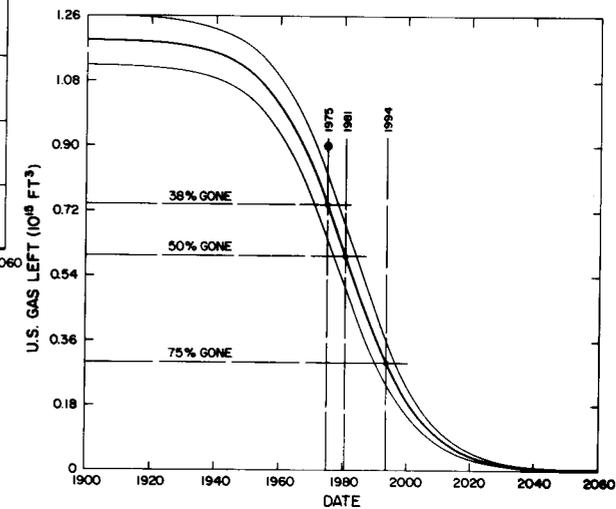
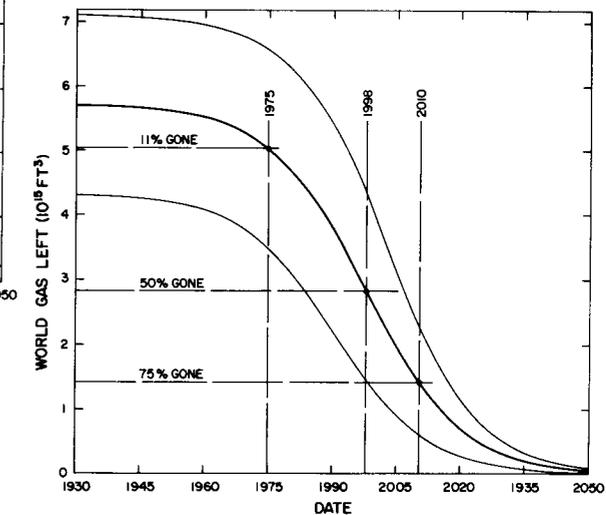


Figure 55



COAL

United States: Data prior to 1868 are from the Department of Commerce (U. S. Department of Commerce, 1960); the rest are from *Minerals Yearbook*. A single-peak fit gives a ridiculously low prediction. This is expected because of the substitution of oil and gas for coal from the 1920's on. Our prediction is a three-peak fit which peaks at ~2035. As for all fits to production data which have not yet peaked, we must caution that our prediction may be pessimistic since we do not allow for any asymmetry. However, our Q(1975) value is in good agreement with Parker's estimate (●) (Parker, 1975) of economically recoverable United States coal but is considerably larger than the estimate (✕) of Schmidt and Hill (Schmidt and Hill, 1976).

**Peak date — 2040, Half date — 2035,
19% gone in 1975.**

World: We found no data for 1945-1946. The dashed line is a two-peak fit to the data. The solid line is a single-peak fit, which we use for our prediction because it is more optimistic. We predict peaking at ~2052. Our Q(1975) value is about 1.5 times larger than Parker's estimate (●) (Parker, 1975) of economically recoverable World coal.

**Peak date — Half date — 2052,
15% gone in 1975.**

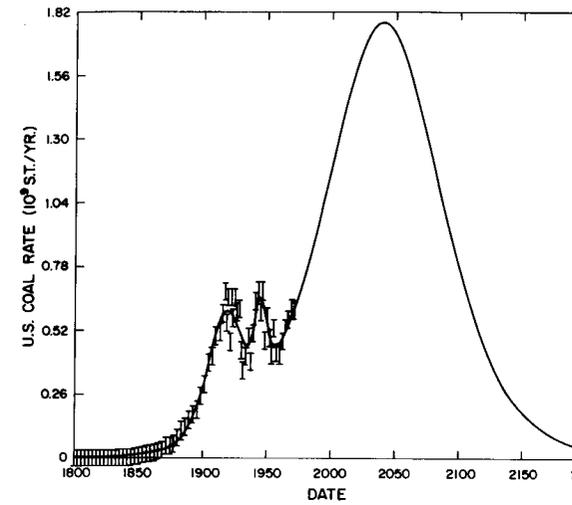


Figure 56

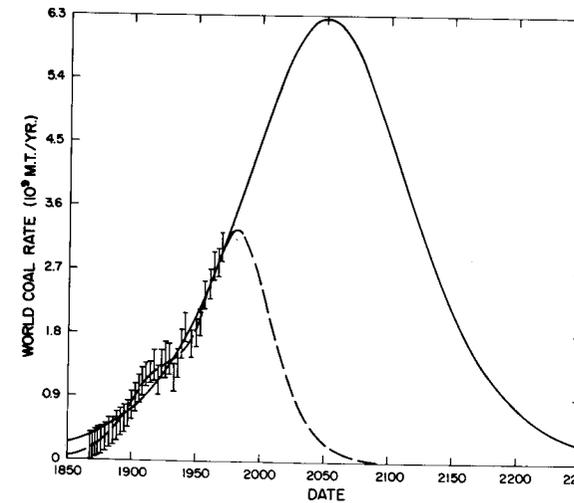
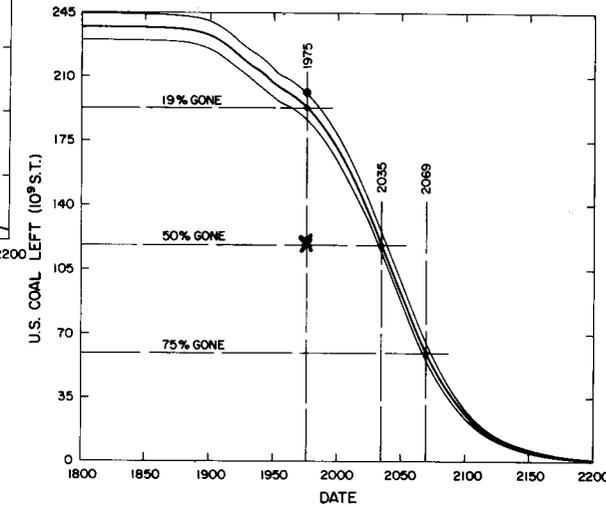
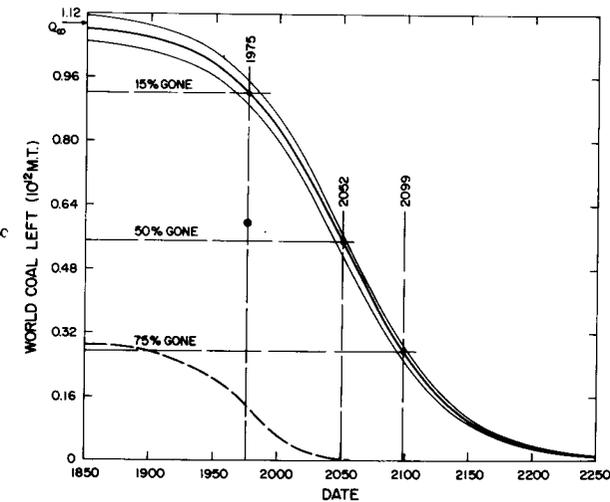


Figure 57



MINERAL FUELS

United States: This is crude oil, natural gas, and coal combined in units of energy equivalent (kWh). The data prior to 1900 are from the Department of Commerce (U.S. Department of Commerce, 1960), the data for 1900-1923 are from Spencer (Spencer, 1970), and the data after 1923 are from *Coal Facts 1974-75* (Nat. Coal Assoc., 1975). The dashed curve is a two-peak fit and the solid curve is a single-peak fit. We use the latter for our prediction because it is more optimistic. We predict peaking at ~1991. However, as for all fits to production data that have not yet peaked, we must caution that our prediction may be pessimistic since we do not allow for asymmetry. The Q_{∞} sum (●) of crude oil, natural gas, and coal value (3.27×10^{15} kWh) is slightly higher than the Q_{∞} (using 1640 kWh/bbl for oil, 0.293 kWh/ft³ for gas, and 7325 kWh/S.T. for coal) from our separate fits to the different mineral fuels.

**Peak date — Half date — 1999,
30% gone in 1975.**

World: We found no World data.

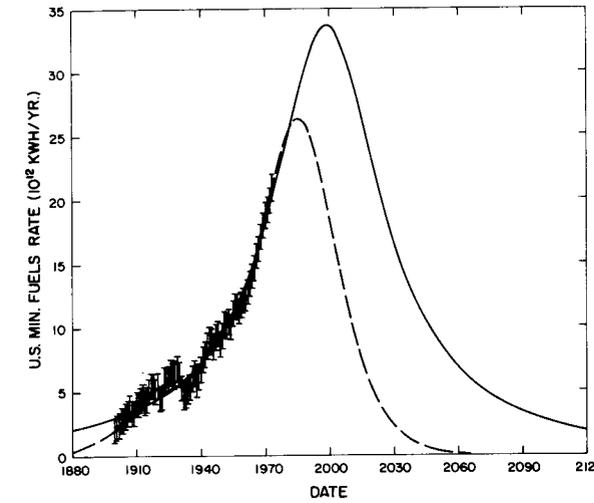
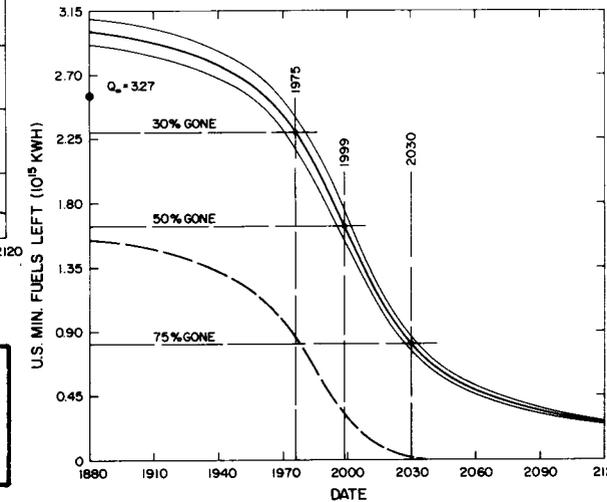


Figure 58



World data on this mineral not available.

PEAT

United States: We found no data prior to 1917 and for 1927-1933, 1950, and 1974. Our fit peaks in 1965. However, we caution that this could be caused by a short-term fluctuation.

**Peak date — 1967, Half date — 1965,
81% gone in 1975.**

World: The *Minerals Yearbook* data are inconsistent in places. We have not been able to reasonably resolve the inconsistency. Therefore, we have no World-peat prediction.

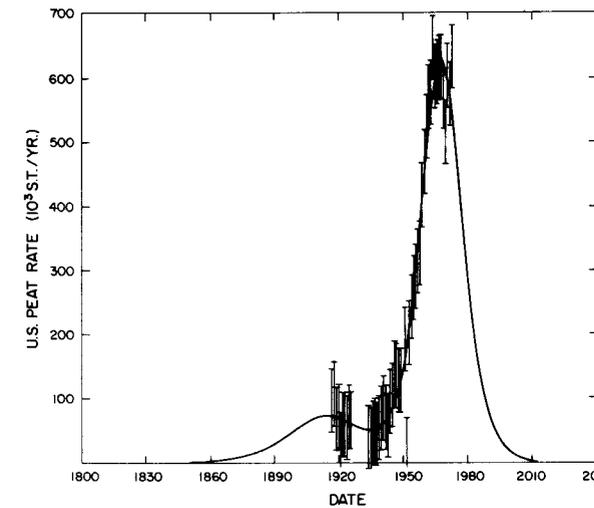
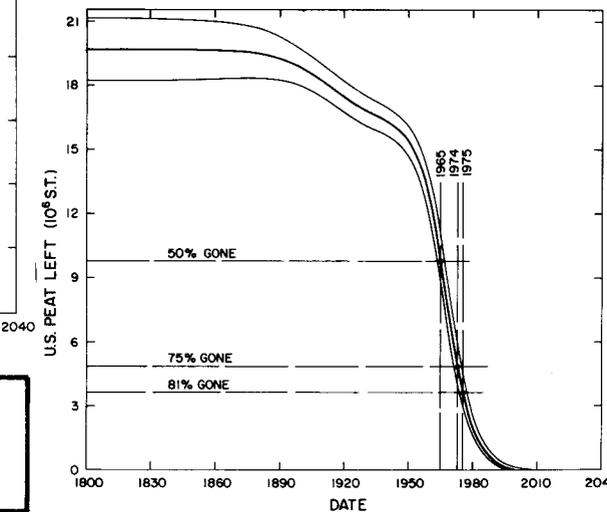


Figure 59



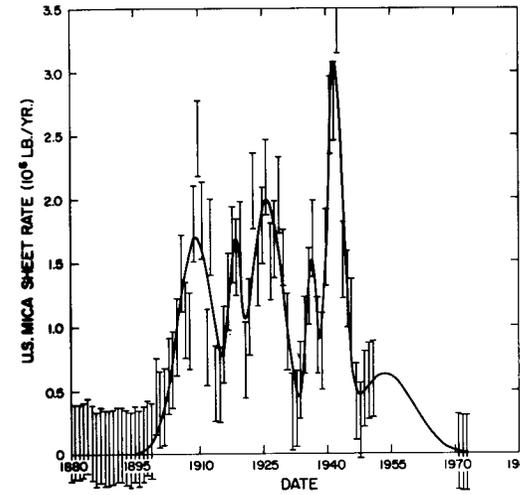
World data on this mineral inconsistent.

III. SELECTED NONMETALS

MICA SHEET

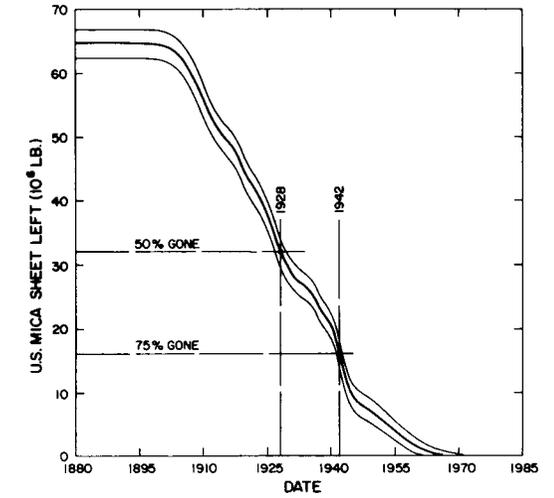
United States: We include this nonmetallic mineral as an example of one that is essentially completely depleted. We found no data for the years 1952-1970 and 1974. Our six-peak fit has a half-date of 1928 and a three-quarter date of 1942.

**Half date — 1928,
100% gone in 1975.**



World data not shown.

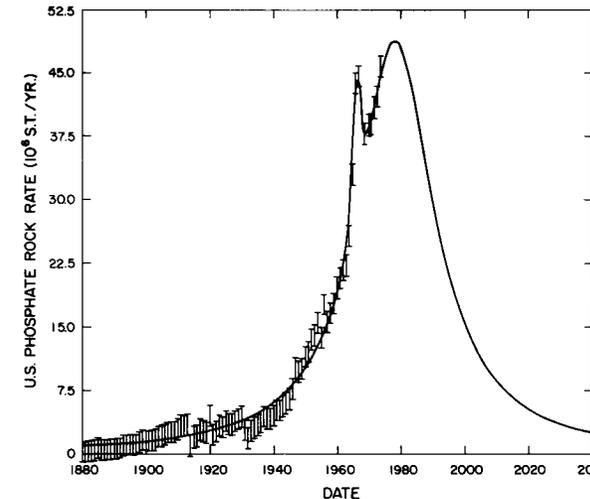
Figure 60



PHOSPHATE ROCK

United States: We include this nonmetallic mineral as an example of one that is not yet depleted. A two-peak fit is more optimistic than a single-peak fit. We caution that our prediction may be pessimistic since we do not allow for any asymmetry.

**Half date — 1978,
45% gone in 1975.**



World data not shown.

Figure 61

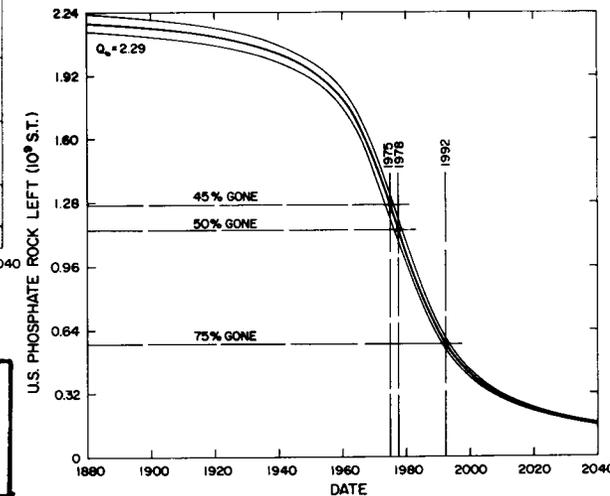


TABLE 2

Mineral Depletion Parameters

t_p = peak date, $t_{1/2}$ = half date, $t_{3/4}$ = three-quarter date. Where a peak date is not given, it is the same as the half date (symmetric fit) or cannot be determined. Maximum uncertainty is indicated by the numeral 4 and maximum certainty is indicated by the numeral 1.

Resource	United States				World					
	Uncertainty Level	t_p	$t_{1/2}$	$t_{3/4}$	% gone in 1975	Uncertainty Level	t_p	$t_{1/2}$	$t_{3/4}$	% gone in 1975
A. Metals										
Antimony	4	2000	1978	2032	48	4	---	2021	2054	20
Arsenic, White	-	---	---	---	--	2	1934	1992	2056	41
Bauxite	3	1968	1966	1981	66	4	---	1993	2005	15
Beryl	3	---	1960	1968	93	2	---	1959	1968	84
Bismuth	-	---	---	---	--	4	---	1983	1999	36
Cadmium	2	1957	1972	1998	54	3	---	1978	1992	44
Chromite	2	---	1955	1958	100	3	---	1981	1996	40
Cobalt	-	---	---	---	--	3	---	1980	1994	41
Copper	3	2020	2017	2048	20	3	---	1988	2014	34
Gold	1	---	1916	1946	90	3	---	2033	2082	16
Iron Ore	2	---	1962	1999	62	3	---	1992	2010	26
Lead	3	1925	1958	2010	60	4	---	2033	2063	16
Magnesium	4	---	2004	2033	20	4	---	1995	2012	23
Manganese Ore	2	---	1955	1961	90	3	---	1973	1989	55
Mercury	1	1870	1916	1976	75	4	---	2038	2076	17
Molybdenum	4	---	2020	2042	10	4	---	2014	2030	6
Nickel	2	---	1972	1986	58	3	---	1980	1995	40
Niobium-Tantalum	3	---	1957.5	1958.5	100	3	---	1970	1975	75
Platinum Group	1	1923	1941	1965	82	3	---	1973	1983	57
Selenium	2	---	1961	1972	79	-	---	---	---	--
Silver	1	1908	1938	1987	70	3	1945	1978	2044	49
Tellurium	2	---	1964	1975	75	-	---	---	---	--
Tin	-	---	---	---	--	4	---	2146	2233	11
Titanium (Ilm.)	2	---	1966	1975	75	4	---	1986	1997	26
Titanium (Rut.)	3	---	1958	1967	90	3	---	1970	1976	70
Tungsten Ore	3	---	1962	1977	72	3	---	1971	1989	57
Vanadium	3	---	1968	1977	69	4	---	1983	1989	18
Zinc	2	1943	1968	2016	54	4	---	1994	2014	27
B. Mineral Fuels										
Crude Oil	3	---	1984	2006	40	4	---	2034	2050	1.5
Natural Gas	3	---	1981	1994	38	4	---	1998	2010	11
Coal	4	2040	2035	2069	19	4	---	2052	2099	15
Peat	3	1967	1965	1974	81	-	---	---	---	--
Mineral Fuels	4	---	1999	2030	30	-	---	---	---	--
C. Selected Nonmetals										
Mica Sheet	1	---	1928	1942	100	-	---	---	---	--
Phosphate Rock	4	---	1978	1992	45	-	---	---	---	--

Chapter 6 CONCLUSION

The production data for a mineral are a rich source of information, not only of the past history of the mineral, but also of its future. We have used reasonable assumptions about mineral depletion in this monograph to roughly predict some parameters of mineral depletion in both the United States and the World, with emphasis on metals and mineral fuels. The results are shown in the Figures of Chapter 5 and are summarized in Table 2. The "uncertainty level" column of Table 2 is our somewhat subjective assessment of the reliability of our predictions. Maximum uncertainty is indicated by the numeral 4 and maximum certainty is indicated by the numeral 1.

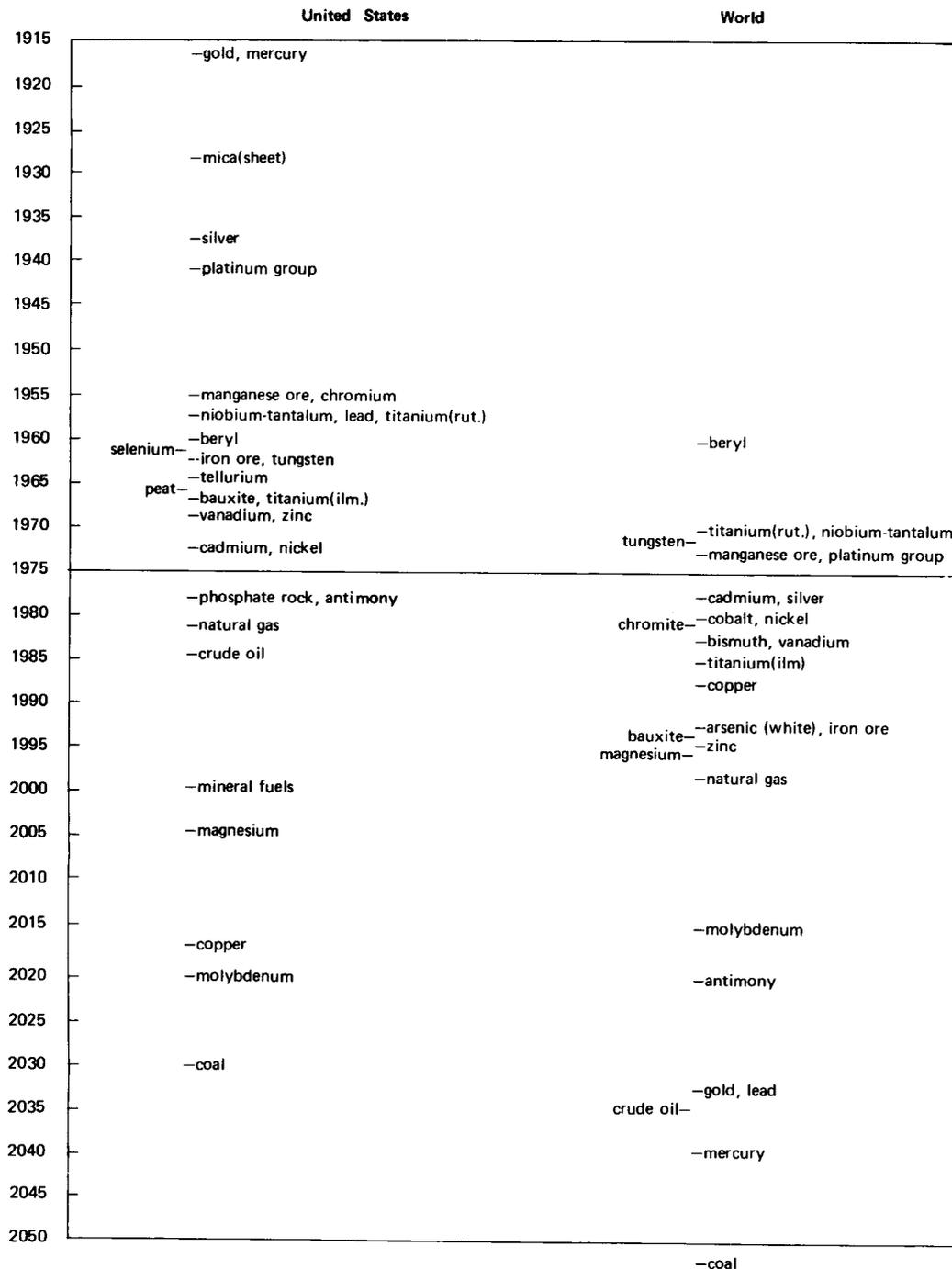
Table 3 is a plot of our depletion half dates for the various minerals for both the United States and the World. From a brief study of this table we can summarize it with the following succinct statements:

1. Most United States metals have already peaked; only four have not.
2. Most World metals have not yet peaked; only eight have already peaked.
3. Neither crude oil nor natural gas have yet peaked for both the United States and the World, although both crude oil and natural gas will peak very soon for the United States.
4. Thus, we conclude that the United States is in a much more severe "metals crisis" than "fuels crisis." That is, most metals have already peaked in production, whereas no mineral fuel has yet peaked. However, the crucial importance of energy to all motions and transformations of matter, including mining and processing of metals, and the lack of a wide variety of mineral fuels and other presently available energy substitutes make the early stages of an energy crisis more traumatic than similar stages of a metals crisis.
5. The World's mineral crisis will be about twenty years after the United States' mineral crisis. Rough dates for crisis recognition are 1970 for the United States and 1990 for the world.

We do not claim much precision for our predictions; rather we view them as rough estimates. However, after completing this work we have come to believe that one could improve the predictions by the following procedure:

TABLE 3

DEPLETION HALF DATES



1. Find an average n value (\bar{n}) for the fits to Eq. (6) for all of the minerals that are well past their peak.
2. Fix $n = \bar{n}$ and fit all other mineral data to Eq. (6) by varying the other three parameters.

This will prescribe an identical asymmetry skewed toward large times for all minerals that have not yet peaked, which should be more accurate than the identical symmetry that we have assumed in this work. There are three categories of asymmetry for the United States and World metals that are well past peaking, as shown in Table 4. From Table 4 we conclude that one could confidently predict an *upper limit* on future production

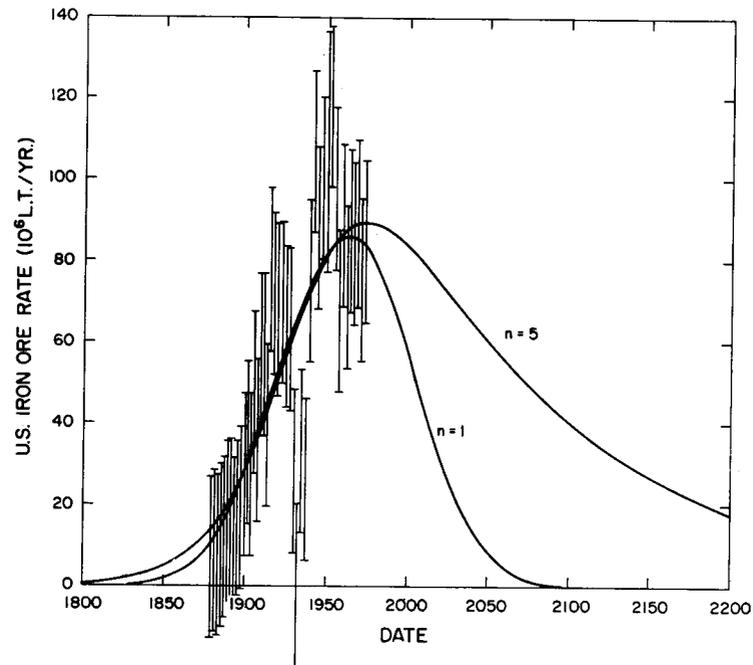
TABLE 4
Asymmetry Categories

Metal	Symmetric ($n = 1$)	Asymmetric $n = 5$	Asymmetric $n = 14$
Beryl (World)	$n = 1$		
Gold (U. S.)	$n = 1$		
Cadmium (U. S.)		$n = 5.0 \pm 2.2$	
Lead (U. S.)		$n = 6.4 \pm 3.1$	
Silver (U. S.)		$n = 6.1 \pm 1.9$	
Silver (World)		$n = 3.5 \pm 4.1$	
Zinc (U. S.)		$n = 4.7 \pm 2.6$	
Arsenic, white (World)			$n = 13.2 \pm 6.4$
Mercury (U. S.)			$n = 14 \pm 12$
Platinum Group (U. S.)			$n = 14.9 \pm 6.4$

(n is the asymmetry parameter in Eq. (6) of Chapter Two.)

of a mineral by fixing $n = 14$ in a fit of Eq. (6) to the production data. However, one could argue that United States mercury and platinum group have such large asymmetries because they peaked so early in the development of the country. (We can conjure no similar argument for World white arsenic.) Therefore, a more realistic prediction would be a fit using Eq. (6) with n fixed at 5. In Figure 62 we show a comparison of United States iron-ore predictions for $n = 1$ and $n = 5$. There is a difference of fifty-nine years in the $t_{1/2}$ values. Note, however, that the peak dates are not very different (~ 10 years). (Peaking of an important mineral can be a very traumatic event to a society.) Also, note that the 1960 reserves estimate (●) (Frasché, 1962) is in much better agreement with $n = 1$ than with $n = 5$. Also plotted is a 1973 reserves estimate (●) (Brobst and Pratt, 1973) that lies about halfway between the $n = 1$ and $n = 5$ curves. In a nontechnical version of this work intended for the general reader⁷ this optimistic prediction method (with $n = 5$) is used for many United States metals.

Figure 62



About one-half of the United States fits are for the logistic curve or its generalized version, the Verhulst curve. Most of the remainder are for the error function curve. However, about one-half of the world fits are for the arccotangent curve. Thus, it appears that quite different socio-technical forces are at work in the two cases. The infancy and prodigy of the United States on the World minerals production scene are probably the main reasons for the difference.

There is little room for argument about most of the United States metals having already peaked. The peak date (see Table 2) is probably the most significant depletion parameter because it signals the end of growth in production rate, which undoubtedly is a traumatic event for a society — particularly when many different important metals peak in the period of one or two decades as has occurred for the United States. Also, the peak date is the most reliable of the predicted depletion parameters for minerals that have not yet peaked.

It appears that fruitful avenues for future work would be to:

1. Do fits to nonmetallic minerals to ascertain whether there is further evidence for universality of the categories $n = 1$, $n = 5$, and $n = 14$.
2. Refit the metallic minerals using $n = 5$ and compare the predictions to the $n = 1$ fit given here. (An example of iron ore is given above.)

3. Check for consistency of predictions using different data-cutoff dates, similar to what we did for United States crude oil in Chapter Five.
4. Do fits to mineral production data for other countries of the World and compare the relative depletion positions of the different countries.
5. Do fits to specific regions within the United States to aid in planning for the future for those regions.
6. Study the relationships between short-term mineral production fluctuations and social events such as wars and depressions.
7. Study the relationships, if any, between the fact that most major United States metals have peaked in the last fifteen years and the recent economic difficulties.

REFERENCES

- Brobst, D.A., and W. P. Pratt, Ed., 1973. Geological Survey Professional Paper 820, U. S. Gov. Printing Office, Washington, D.C.
- Cloud, P., 1973. "Mineral Resources in Fact and Fancy" in H. E. Daly, Ed., *Toward a Steady State Economy*, W. H. Freeman and Co., San Francisco, p. 50.
- Cook, E., 1975. *Tech. Rev.*, **77**, No. 7, p. 15.
- Elliott, M. A., and H. R. Linden, 1968. *J. Pet. Tech.*, February, pp. 135-141.
- Frasché, D. F., 1962. *Mineral Resources, A Report to the Committee on Natural Resources of the National Academy of Sciences - National Research Council*, Pub. 1000-C, Nat. Acad. of Sci., Nat. Res. Council, Washington, D.C.
- Gabel, M., Ed., 1975. *Energy, Earth and Everyone*, Earth Metabolic Design, Inc., Box 2016 Yale Station, New Haven, Conn., 06520, p. 56.
- Hubbert, M. K., 1969. "Energy Resources" in Preston Cloud, Ed., *Resources and Man, Committee on Resources and Man of the National Academy of Sciences - National Research Council*, W. H. Freeman and Co., San Francisco, pp. 157-242.
- Lasky, S. G., 1951. *Eng. and Min. J.*, **152**, No. 8, p. 60.
- Lasky, S. G., 1955. *Eng. and Min. J.*, **156**, No. 9, p. 94.
- National Academy of Sciences - National Research Council, 1975. News Report XXV, No. 2, February.
- National Advisory Council for Development Cooperation - Netherlands, 1975. Report #53, "Recommendations on the Future Availability of Metallic Minerals," The Hague, Netherlands.
- National Coal Association, 1975. *Coal Facts 1974-75*, Washington, D. C.
- Park, C. F., Jr., and M. C. Freeman, 1968. *Affluence in Jeopardy*, Freeman, Cooper and Co., San Francisco.
- Parker, A., 1975. *Energy Policy*, March, p. 58.
- Schmidt, R. A., and G. R. Hill, 1976. *Annual Review of Energy*, **1**, p. 37.
- Spencer, V. E., 1970. "Raw Materials in the United States Economy," Working Paper 30, U. S. Dept. of Comm.
- Steidle, E., 1952. *Mineral Forecast 2000 A. D.*, The Pennsylvania State College, State College, Pa.
- United Nations, 1974. *Statistical Yearbook 1973*, 25th Issue, New York.
- U. S. Department of Commerce, 1960. *Historical Statistics of the United States, Colonial Times to 1957*.
- U. S. Department of Commerce, 1965. *Historical Statistics of the United States, Continuation to 1962 and Revisions*.
- U. S. Department of Interior, 1973-1974. "Mineral Industries Surveys," Bur. of Mines, Washington, D. C.
- U. S. Government Printing Office, 1832-1972. *Minerals Yearbook*.
- U. S. Government Printing Office, 1883-1931. *Mineral Resources in the United States*.

INDEX

- antimony 14
- arsenic 16, 88
- asymmetry 5, 11, 85
- asymmetry parameter (n) 6, 11, 85
- background behavior** 4
- bauxite 18
- beryl 20
- bismuth 22
- cadmium** 24
- chromite 26
- coal 76
- cobalt 28
- Cook 1, 2, 36, 70
- copper 30
- crisis recognition 2, 83
- data** 9
- depletion parameters 82
- depletion theory 3
- energy crisis** 87
- error channel 11
- error function curve 6, 86
- fitting procedure** 11
- fuels crisis 2, 83
- gas** 1, 74, 83
- Gaussian function 6
- gold 32
- Gompertz curve 6, 20
- half date** 1, 5, 82, 83, 84
- Hubbert 1
- ilmenite** 60
- inverse cotangent curve 7, 88
- iron ore 34, 85
- k 1, 5
- Lasky** 1
- lead 36
- least-square fit 11
- logistic curve 5, 88
- Magnesium** 38
- manganese ore 40
- mercury 42, 85
- metals crisis 2, 83
- mica sheet 80
- mineral fuels 78
- minerals crisis 83
- Minerals Yearbook* 9, 14, 36, 70, 72, 74, 76
- molybdenum 44
- n 5, 11, 85
- nickel 46
- niobium 48
- nonmetallic minerals 80, 82, 86
- ocean** 38
- oil 1, 72, 83
- peak date** 1, 82, 86
- peat 78
- phosphate rock 80
- platinum group 50, 85
- production rate (P(t)) 1, 3, 4
- P(t) 1, 3, 4, 5
- Q(t) 1, 5, 9
- Q_∞ 2, 5
- rate constant (k)** 1, 5
- recycling 1
- reserves estimates 1, 9
- rutile 62
- selenium** 52
- short-term fluctuations 4
- silver 54
- t_{1/2} 2, 5, 11, 82
- t_{3/4} 2, 82
- t_p 2, 82
- tantalum 48
- tellurium 56
- three-quarter date 1, 82
- tin 58
- titanium 60, 62
- tungsten ore 64
- uncertainty level** 82, 83
- uranium oxide 66
- vanadium** 68
- variable decay-rate model 5
- Verhulst curve 5, 86
- zinc** 70