Part I. Major Ice Ages Earth-States Transitions

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Abstract

A double-sigmoid Earth-states transitions model is used to fit Antarctica temperatures (determined by deuterium measurements in Vostok ice cores) using calculated summer North-Pole solar insolation. Fits to the last four Major Ice Ages (-422 to -11 kiloyears before present by solar insolation peaks) are used to predict the next two Major Ice Ages (-11 to 230 kiloyears from present by solar insolation peaks). Double-sigmoid transitions functions within a Major Ice Age can be interpreted as turning on and off of Earth states, possibly including parts or all of the Great Ocean Conveyor Belt. The model's equation also contains a term with a constant factor times the summer North-Pole solar insolation and a constant-in-time term. Two double sigmoids for each Major Ice Age are found to provide a good fit to tuned Antarctica temperature data. Fits are done with the same double-sigmoid parameters for all four Major Ice Ages and with different parameters for different Major Ice Ages. In Part II. some of the most prominent fine structure in Major Ice Ages are well fitted with one or two more double sigmoids for each Major Ice Age.

Introduction

Possibly the most important single measurement of temperatures of the previous four Major Ice Ages (defined below) are Antarctica atmospheric temperatures back 422 kiloyears, determined from deuterium concentrations in Vostok ice cores[1]. The data can be downloaded[2] from the Internet.

In this paper a model for the Major Ice Ages is used to fit the measured Antarctica temperatures of the last four Major Ice Ages (defined below) using the calculated summer North-Pole solar insolation (defined below). The model is designed such that it can be used to predict future Major Ice Ages.

The model involves double-sigmoid transitions functions in the time regions of each Major Ice Age, which can be interpreted as the turning on and off of earth states, possibly including parts or all of the Great Ocean Conveyor Belt[3] (GOCB). The model equation also contains a fraction of the North-Pole solar insolation on 21 June and a constant in time.

A "Major Ice Age" is defined as a period of time, maximum peak to peak temperatures (actually the nearest peaks of summer North-Pole solar insolation; see below) of the order of 115 kiloyears, during which the Earth's mean atmospheric temperature is $4C^\circ$ to $9C^\circ$ colder than at present and ice sheets and glaciers extend much beyond their present boundaries. "Major Interglacials" are warm periods of 5-10,000 years duration at the boundaries between the Major Ice Ages. In this paper the term Last Major Ice Age (LMIA) will refer to the Major Ice Age that the Earth just experienced; the term Penultimate Major Ice Age (PMIA) will refer to the Major Ice Age before the Last Major Ice Age, and the term Next Major Ice Age (NMIA) will refer to the Major Ice Age that the Earth is poised to enter. Similarly, the term Current Major Interglacial refers to the Major Interglacial the Earth is experiencing, Last Major Interglacial refers to the Major Interglacial at the beginning of the Last Major Ice Age (traditionally called the Emian Interglacial), at about 120,000 years ago, and Next Major Interglacial refers to the Major Interglacial at the end of the Next Major Ice Age.

Within a Major Ice Age are several "Minor Ice Ages", also called stadials: periods of time, of the order of 10,000 to 20,000 years, during which the temperature is about $2C^\circ$ colder than the warm periods ("Minor

Interglacials", also called interstadials) around them. Within the Minor Ice Ages and Minor Interglacials are "Little Ice Ages" and "Little Interglacials" with durations of one to two thousand years and amplitudes of about $1C^{\circ}$ in size, some periodic and some random. The last Little Ice Age temperature minimum occurred at about 1500-1850 BC[3].

The Vostok Antarctica ice core deuterium-determined atmospheric temperatures[1] relative to the current mean temperature for the last 424 kiloyears are shown in Figure 1. Note the similar behavior of the Major Interglacial at about -420 kiloyears before present and the Major Interglacial the Earth is now experiencing. Also note the similar behavior of the Major Interglacial at about -320 kiloyears before present and the Major Interglacial at about -235 kiloyears before present and the one at about -125 kiloyears before present.

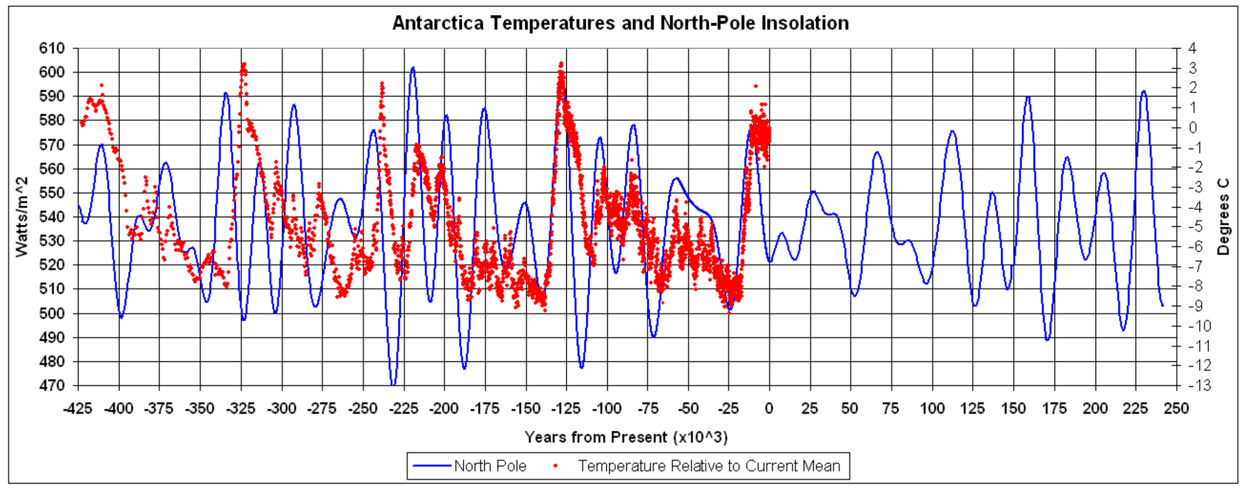


Figure 1. Vostok Antarctica atmospheric temperatures[1] (right axis) relative to the current mean temperature, as determined from deuterium in ice cores, and summer North-Pole insolation (left axis) calculated using the Berger code[3].

Solar Insolation and Antarctica Temperatures

One can compare solar insolation calculations, which are very accurate, with the Antarctica temperature data described above and get a high correlation. *Solar insolation* is the amount of power per area (energy/time/area in units Watts/m^2) delivered by the Sun to the Earth's upper atmosphere, a function of the latitude and the time of year. Insolation varies widely over time and latitude due to changes in the Earth's orbit and tilt caused by effects of the orbits of the other planets and the moon, which is manifested in several periods. (*Solar irradiance* is the power *per solar surface area emitted* by the Sun, which varies very little with time compared to insolation.)

Solar insolation calculation computer programs of Berger, *et al*.[4] and Laskar, *et al*.[5] can be downloaded from the Internet[6]. The calculations stretch from $-20x10^6$ to $+10x10^6$ years for the Laskar code and $-1x10^6$ to $+1x10^6$ years for the Berger code with much higher accuracy than any measurements of Earth temperatures into the distant past. The summer (21 June) North-Pole insolation curve versus time, along with Antarctica atmospheric temperatures, is shown in Figure 1. (Northern latitude insolation is expected to have the largest effect on atmospheric temperatures, even at southern latitudes, because most of the land mass, which can support formation of large ice sheets, is at northern latitudes[3]. See below for a correlation study that corroborates this.)

A quick study of Figure 1. makes it clear that, relative to the situation at the Major Interglacials, some Earth processes (e.g., melting ice) are taking solar energy away from heating the atmosphere during the insolation peaks and some Earth processes (e.g., forming ice) are adding heat energy beyond that supplied by solar energy to the atmosphere during the insolation valleys. At the Major Interglacials the temperature is higher relative to insolation than it is at the insolation peaks between the Major Interglacials. That is, the insolation is amplitude modulated in a specific way to produce the temperatures over the course of a Major Ice Age. These are the clues that led to the model used in this work. An equation was sought that would model these Earth-state processes.

Some features of summer North-Pole insolation for three of the Major Ice Ages considered in this paper are given in Table 1.

Table 1. Some features of the summer North-Pole insolation (MIA = Major Ice Age)

Feature	Penultimate MIA	Last MIA	Next MIA
Average of 1000-year values over the MIA (Watts/m^2)	537	536	532
Median of 1000-Year values over the MIA (Watts/m^2)	537	535	527
Average peak (Watts/m^2)	582	576	561
Average valley (Watts/m^2)	495	496	516
Average peak – average valley	87	80	45
Length (peak to peak in kiloyears)	116	117	123

Thus, the average energy delivered to the North-Pole upper atmosphere over the years of a Major Ice Age will be slightly less during the Next Major Ice Age than it was for the Last and the Penultimate Major Ice Ages. The NMIA will be about 5% longer than the LMIA. Since the difference between the average peak size and the average valley size for the NMIA is about half that of the LMIA, it is expected that the Antarctica temperature variations will be less during the NMIA than they were for the LMIA.

Visually, in Figure 1. there appears to be a high correlation between Antarctica atmospheric temperatures and summer North-Pole insolation, with perhaps a time lag.

Correlation calculations were done by using only those temperature data that are at or near a thousand-year mark relative to the present, since those times are when the insolation calculations are usually done, with the results given in Table 2. for the Last Major Ice Age (-128 to -11 kiloyears before the present) and over the entire range for which the temperatures are available (-424 to 0 kiloyears before present). These correlations corroborate that northern summer insolation is more important than southern summer insolation with regard to Antarctica temperatures. Then correlation calculations were done for all dates for which the Antarctica temperatures are available and for recent mean Earth temperatures reported in *Climate Change 2001: The Scientific Basis*[7], the results of which also are shown in Table 2.

Table 2. Correlation coefficients between various insolation calculations and Antarctica temperatures for 1000-year dates and all dates.

Item	Correlation Coefficient	Temperature Lag
	1000-year dates	**Last Major Ice Age**
Summer North-Pole insolation	**0.573**	3000 years
Summer South Pole insolation	**0.288**	12,000 years
Summer 65°N insolation	**0.583**	3000 years
Summer insolation averaged over latitudes	**0.500**	1500 years
		-425,000 to 0 years
Summer North-Pole insolation	**0.305**	5000 years
Summer South-Pole insolation	**0.219**	12,000 years
Summer 65°N insolation	**0.293**	5000 years
Summer insolation averaged over latitudes	**0.339**	1500 years
	All Dates	**Last Major Ice Age**
Summer North-Pole insolation	**0.583**	5700 years
Summer 65°N insolation	**0.563**	5700 years
Summer insolation averaged over latitudes	**0.137**	5800 years
		-425,000 to 0 years
Summer North-Pole insolation	**0.470**	4700 years
Summer 65°N insolation	**0.450**	5000 years

Since the summer North-Pole insolation has the highest correlation, I use it in all that follows. I also add the recent mean Earth temperatures reported in *Climate Change 2001: The Scientific Basis*[7], excluding the last 1000 years, to the Antarctica temperatures for fitting purposes.

In all that follows, a Major Ice Age is defined as the peak-to-peak period of the summer North-Pole insolation for the insolation peaks that are the closest to the Major Interglacials for the approximately 115-120 kiloyear period, perhaps shifted by a few thousand years into the future.

Tuning the Three Penultimate Major Ice Ages to Insolation

It is seen in Figure 1. that the maxima and minima of the three Penultimate Major Ice Ages are not all in "tune" with the relevant summer North-Pole insolation maxima and minima. That is, the time intervals between the temperature maxima/minima are not the same as the time intervals between the nearby insolation maxima/minima.

Probably the timing of the temperature measurements is less accurate than the measurement of the temperature through the deuterium content of the ice, because yearly ice layers can be melted or distorted by movement and drifting of the original snow. So it may be in order to tune the times of the temperature data with the times of the insolation.

Therefore, all four of the Major Ice Ages were "tuned" by linearly shifting the maxima and minima backward in time one by one such that the time intervals between the temperature maxima/minima and the time intervals between the insolation maxima/minima are the same. The tuning equation used is

$$t_{new} = \frac{t_{I_2} - t_{I_1}}{t_{T_2} - t_{T_1}}\left(t_{old} - t_{T_1}\right) + t_{T_1}, \text{ where}$$

t_{I_i} = time of i^{th} insolation maximum/minimum and

t_{T_i} = time of i^{th} temperature maximum/minimum for the previous interval.

The tuning was done by starting at the Last Glacial Maximum, the temperature minimum just before the fast rise to the Current Interglacial. There the insolation minimum and the temperature minimum were made to occur at identical times. (Thus, no time lag is allowed. After the fitting is done, a time lag can be inserted if desired.) There are several time regions for which judgment calls had to be made as to which temperature maxima and minima should be associated with which insolation maxima and minima.

The result of the tuning is shown in Figure 2. (The Excel file used to tune the times can be downloaded from the Internet[8].)

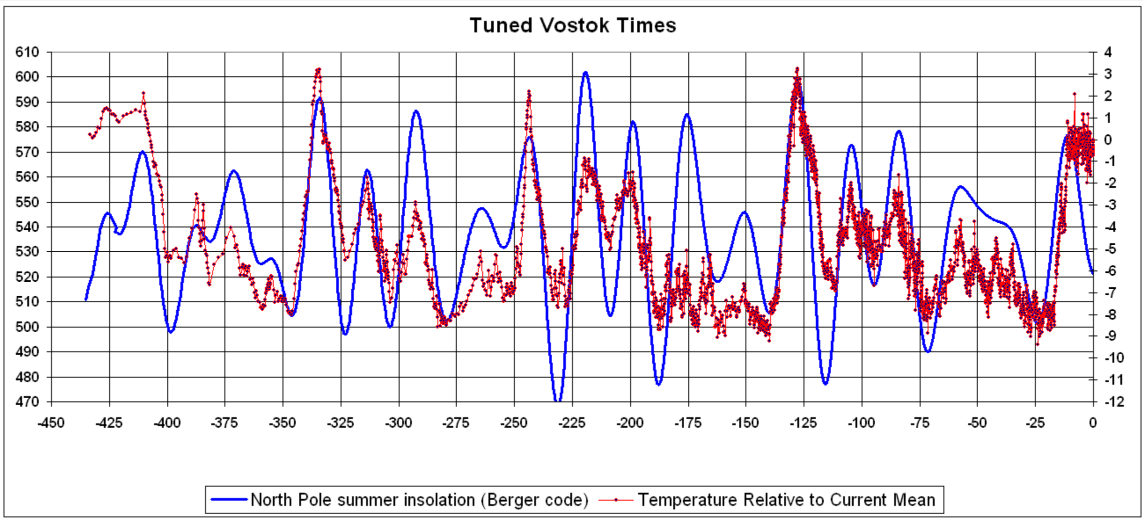


Figure 2. **Tuned** Antarctica temperature data and summer North-Pole insolation. The time region of the most dubious of the tuning decisions is -180 to -140 kiloyears.

Transitions Model for Calculating Temperatures from Insolation

The procedure used to fit the Antarctica temperature data is as follows:

Plot the temperature data and the summer North-Pole insolation curve on the same graph. Adjust the temperature scale such that the insolation minima just before the Last Major Interglacial and the Current Major Interglacial and the insolation maxima of the Last and Current Major Interglacials approximately match the measured temperatures, as shown in Figure 1. (It is either fortuitous or important that this is possible to do all four of these near matches together.) This makes it easier to visually compare the fits to the temperatures with the summer North-Pole insolation.

Referring to Figure 1., a model for fitting the relative Antarctica temperatures for the Last Major Ice Age must have the following features:

1. The temperature as shown in Figure 1. must be lower than the insolation at times of insolation maxima away from Major Interglacials.
2. The temperature as shown in Figure 1. must be higher than the insolation at times of insolation minima away from the valleys just before Major Interglacials (glacial maxima).
3. At glacial maxima (insolation minima just before a Major Interglacial), the temperature as shown in Figure 1. must be about equal to the insolation.
4. At insolation maxima of Major Interglacials, the temperature as shown in Figure 1. must be about equal to the insolation.

That is, the model must be some function of time and insolation that changes from the behavior of points 1. and 2. to the behavior of points 3. and 4.

In this work an equation with variable parameters was devised that fits the Antarctica temperatures for the last four Major Ice age reasonably well. The time dependence comes from the time dependence of the summer North-Pole insolation and double sigmoids in time that represent Earth-state changes. This model is probably related to the multiple-state climate model of Paillard[9]. The emphasis of this double-sigmoid model is on predicting the future as well as modeling the past, and it uses Antarctica temperatures rather than Earth ice volume as the data to be fitted.

The equation for the Major Ice Ages is (see Appendix I):

$$T_I(t) = C + F \cdot I(t) + \sum_{i=1}^{N} \sum_{j=1}^{N_{MIA}+1} s_{ij} \frac{1}{2}\left[\tanh\left(\frac{t-\left(c_{1ij}+t_j\right)}{w_{1ij}}\right) - \tanh\left(\frac{t-\left(c_{2ij}+t_j\right)}{w_{2ij}}\right)\right]$$

= temperature in degrees C, where $I(t)$ = calculated North-Pole insolation at time t,

t_j = time of the j[th] Major Interglacial relative to now.

Double-sigmoid parameters: s_{ij} = strength, c_{nij} = position, w_{nij} = width.

N = number of double sigmoids used in the fit.

N_{MIA} = number of Major Ice Ages to be fitted and predicted.

The parameters to be varied to fit the temperature data are $\left(C, F, s_{ij}, c_{nij}, w_{nij}, t_j\right)$. The equation allows the option of using one or more double sigmoids (the i index) for each Major Ice Age. The t_j parameters were started at the values for the Major Interglacials' insolation peaks; they then were allowed to vary after the other parameters were varied for a best fit for the case of using the same sigmoids' parameters for all Major Ice Ages.

For the case of allowing the sigmoids' parameters to be different for the different Major Ice Ages, the t_j parameters were not varied because they are linearly combined with the c_{nij} variable parameters.

Note the use of hyperbolic tangent functions, instead of exponential functions, for expressing the double sigmoids. Doing so makes the equation simpler and alleviates calculational difficulties due to infinite asymptotic properties of exponentials with positive arguments. (See Appendix I.)

The term $F \cdot I(t)$ term is very important, because it provides the correlation of temperature with insolation. F can be considered the temperature "forcing" factor due to the insolation. See the Solar Irradiance Variation section near the end of this paper, which reports fits in which this term is multiplied by a factor containing one or two cosine terms.

The equation was used to fit the Antarctica tuned and untuned temperature data for the last four Major Ice Ages and then used to predict the Antarctica temperatures for the next two Major Ice Ages.

The following fits will be reported below:

1. One double sigmoid for each of the four past Major Ice Ages and only the Last Major Ice Age, using the same five double-sigmoid parameters for the six Major Ice Ages. This predicts the next two Major Ice Ages.
2. Two double sigmoids for each of the four past Major Ice Ages and the Last Major Ice Age, using the same ten double-sigmoid parameters for all six Major Ice Ages. This predicts the next two Major Ice Ages.
3. One double sigmoid for each of the four past Major Ice Ages, with each Major Ice Age having a different set of the five double-sigmoid parameters. For this case a prediction of future Major Ice ages is not possible.
4. Two double sigmoids for each of the four past Major Ice Ages, with each Major Ice Age having a different set of the ten double-sigmoid parameters. For this case a prediction of future Major Ice ages is not possible.

Transitions Model's Fits to Antarctica Temperatures of the Last Four Major Ice Ages

The equation given above for the double-sigmoid transitions model is used to calculate the Antarctica temperatures. The calculated temperatures are then compared to the measured temperatures by a chi-squared minimization procedure, which varies the equation's parameters to achieve a best fit.

The SOLVER procedure of Microsoft's EXCEL spreadsheet[10] was used to search for best fits by minimizing a spreadsheet cell that calculates the square of differences between the measured temperatures and the calculated temperatures. The equation $\chi^2 = \sum_{i=1}^{N} \left(T_{mi} - T_{ci}\right)^2$ yields the measure of the goodness of a fit to the data. No probability can be assigned for goodness of fit because no measurement errors are given for the measured Antarctica temperature data. The implied arbitrary denominator of value 1 in χ^2 is equivalent to an error of 1 degrees C for each temperature data point.

Transitions Model's Fit with One Double Sigmoid with Identical Parameters for each Major Ice Age

The result of the best fit obtained for the one double-sigmoid transitions model to the tuned Antarctica atmospheric temperatures of the last four Major Ice Ages and only the Last Major Ice Age using the same double sigmoid parameters for each Major Ice Age is shown in Table 3. and Figure 3a. This appears to be a unique solution; no other solution was found that fit the data nearly as well as this solution.

Note the rapid onset of the first temperature-increasing transition between the glacial maxima and the Major Interglacials and the slow turnoff over the course of the Major Ice Ages.

In the bottom graph of Figure 3a. it is seen that the main correlation with the summer North Pole insolation is due to the turning on and off of an Earth state in the vicinity of a large insolation maximum, which provides the saw-tooth Major Ice Ages wave. The earth state is probably the Great Ocean Conveyor Belt. However, there is a further correlation (factor of 0.03656) with the insolation *per se*. So, even a very simple model such as this one gives this very interesting information.

A fit was made to the untuned data also, with a similar result, but, of course, a considerably higher chi square.

The prediction for the two Next Ice Ages will be discussed in the conclusion after the two-double-sigmoids fit with constant parameters is presented.

Table 3. Fit parameters for the one double-sigmoid transitions model with the same parameters for each Major Ice Age (MIA). (**Red** indicates variable parameters.) The first line of a table cell is for the fit to all four past Major Ice Ages excluding the last 1000 years. The second line is for the fit to the Last Major Ice Age plus the rise into it, but excluding the last 1000 years. Note that the fit is to tuned data, with no allowance for a time delay between North-Pole insolation and Antarctica temperature. Therefore, the positions (c_{ij}) may have a time delay of a few thousand years. The last row in the table gives the chi squares for the same fits to the untuned data.

s_1 (Degrees C)	c_{11} (kiloyears)	w_{11} (kiloyears)	c_{12} (kiloyears)	w_{12} (kiloyears)	F (Degrees C/(Watts/m^1))	C (Degrees C)
8.495 **12.06**	**-3.680** **-1.134**	**23.69** **5.287**	**13.03** **6.400**	**30.73** **21.40**	**0.03656** **0.03675**	**-25.96** **-25.57**
t_{3PMI} (kiloyears)	t_{2PMI} (kiloyears)	t_{PMI} (kiloyears)	t_{LMI} (kiloyears)	t_{CMI} (kiloyears)	t_{NMI} (kiloyears)	t_{2NMI} (kiloyears)
-429.4 **-429.4**	**-338.7** **-338.7**	**-243.6** **-243.6**	**-130.6** **-132.5**	**-8.995** **-11.00**	112	230
Tuned data Chi Square:	4246 1315	Data:	3272 1937	Parameters:	12	
Untuned Data Chi Square:	5845 2018					

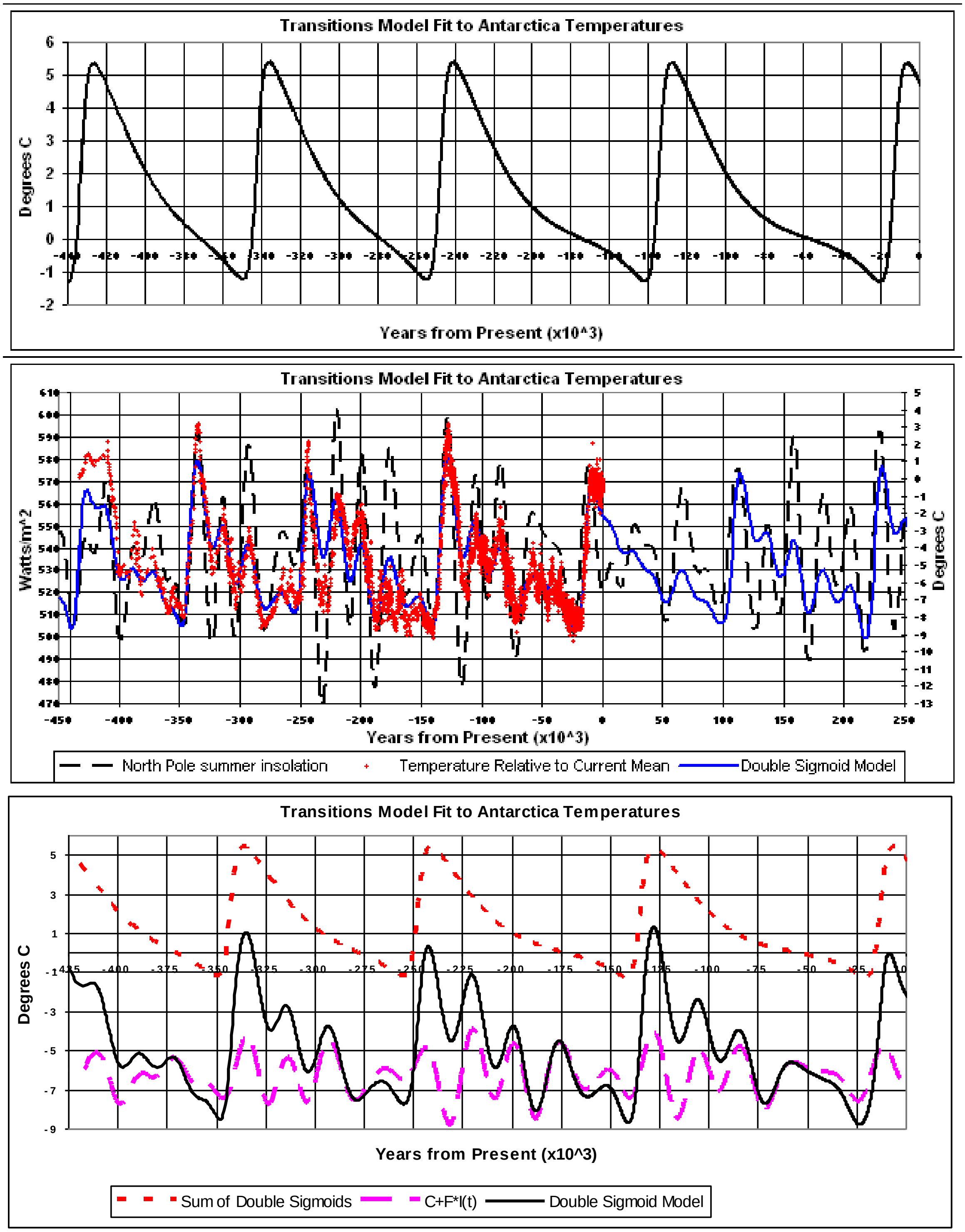


Figure 3a. The fit to the tuned Antarctica temperatures of the last four Major Ice Ages using the transitions model described above with one double sigmoid which has the same parameters for all six Major Ice Ages. The double sigmoids for the Major Ice Ages are shown in the top graph. The components of the fit are shown in the bottom graph.

An interesting question is: How well can the data for the Last Major Ice Age be fitted with solar insolation alone? Figure 3b. shows such a fit. Obviously, the fit is quite bad (χ^2 =10,450 compared to 1315 for fit above with the transitions double-sigmoid); the transitions double-sigmoid is needed to get a decent fit.

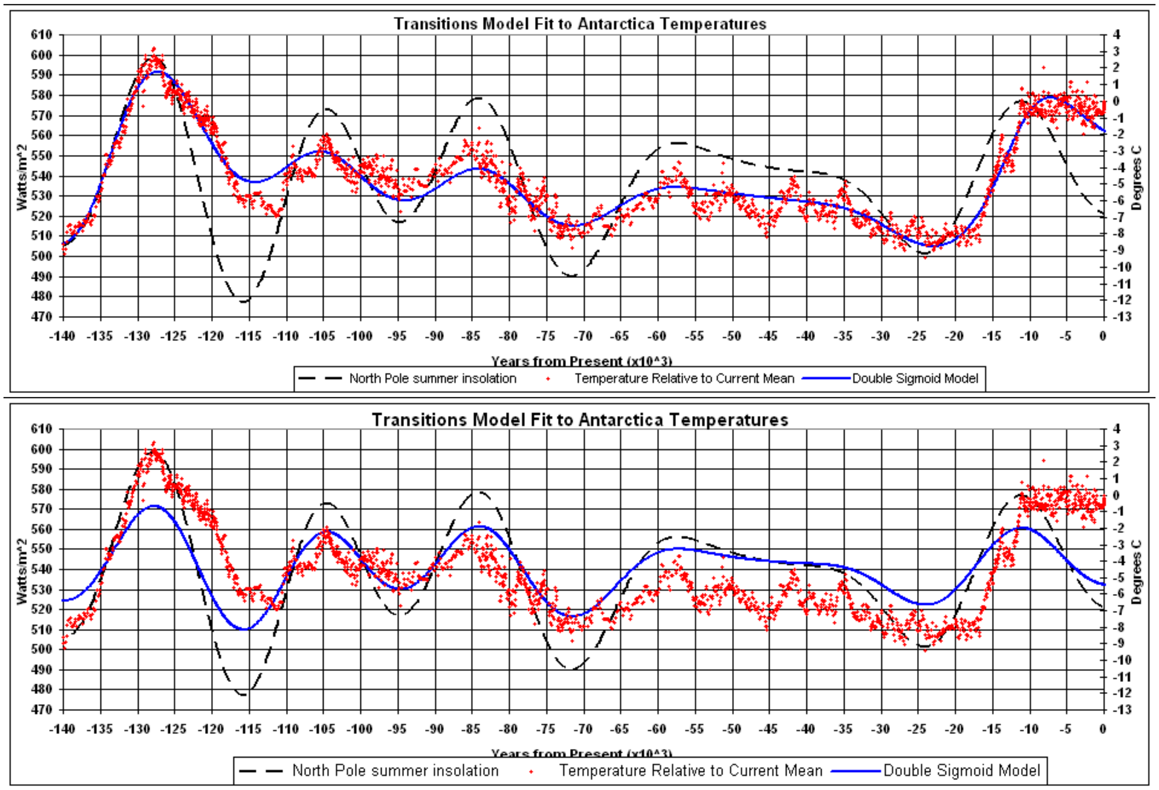


Figure 3b. The top graph is the fit given above and the bottom graph is a fit to the tuned Antarctica temperatures with only the solar insolation and constant term in the model equation.

Another interesting question is: How well can the data for the Last Major Ice Age be fitted without including the solar insolation? Figure 3c. shows such a fit. The fit is not very good (χ^2 =2822 compared to 1315 for fit above with the solar insolation included); the solar insolation is very important to get a good fit.

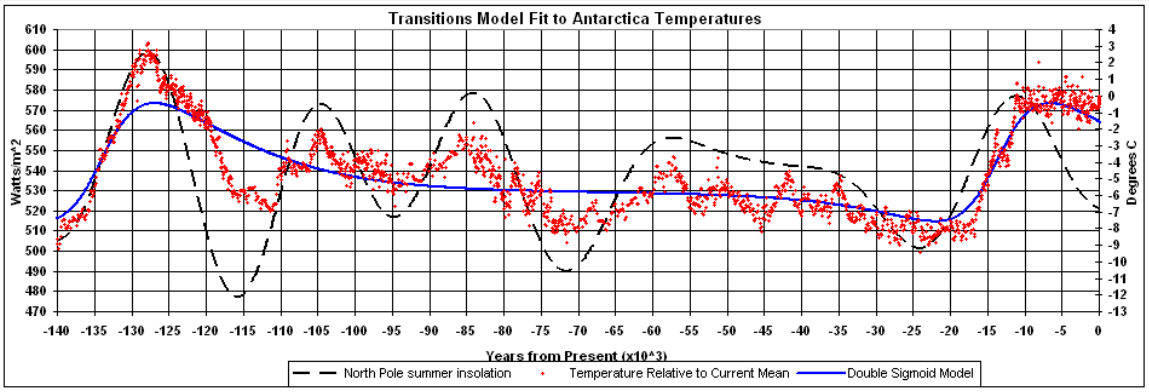


Figure 3c. A fit to the tuned Antarctica temperatures with only the double-sigmoid transitions in the model equation.

Transitions Model's Fit with Two Double Sigmoids with Identical Parameters for each Major Ice Age

The result of the best fit obtained for the two-double-sigmoids transitions model to the tuned Antarctica atmospheric temperatures of the last four Major Ice Ages and only the Last Major Ice Age using the same two double sigmoids for each Major Ice Age is shown in Table 4. and Figure 4. This appears to be a unique solution type; no other solution type was found that fit the data nearly as well as this solution type. Some of the parameters have large uncertainties.

Similar fits were made to the untuned data also, with a similar result, but, of course, a considerably higher chi squares.

This version of the model could be interpreted as two overlapping events of turning on and off of the Great Ocean Conveyor Belt (GOCB), which must correspond to different parts of the GOCB turning on and off. There are three known parts of the GOCB that could be involved: the far northern Greenland Sea part, the more southerly northern Labrador Sea part and the Antarctica part[11]. The fit indicates that the two parts turn on near the same time, but turn off at widely separated times.

Table 4. Fit parameters for the two-double-sigmoids transitions model with the same parameters for each Major Ice Age (MIA). (**Red** indicates variable parameters.) The first line of a table cell is for the fit to all four past Major Ice Ages excluding the last 1000 years. The second line if for the fit to the Last Major Ice Age plus the rise into it, but excluding the last 1000 years. Note that the fit is to tuned data, with no allowance for a time delay between North-Pole insolation and Antarctica temperature. Therefore, the positions (c_{ij}) may have a time delay of a few thousand years. The last row in the table gives the chi square for a fit to the untuned data.

s_1 (Degrees C)	c_{11} (kiloyears)	w_{11} (kiloyears)	c_{12} (kiloyears)	w_{12} (kiloyears)	F (Degrees C/(Watts/m^1))	C (Degrees C)
2.950 **2.936**	**-6.674** **-7.174**	**1.041** **0.919**	**50.88** **56.82**	**10.84** **32.88**	**0.03277** **0.03457**	**-24.64** **-25.71**
s_2 (Degrees C)	c_{21} (kiloyears)	w_{21} (kiloyears)	c_{22} (kiloyears)	w_{22} (kiloyears)		
3.373 **3.666**	**-2.917** **-3.143**	**0.885** **1.494**	**10.35** **10.83**	**2.233** **1.573**		
t_{3PMI} (kiloyears)	t_{2PMI} (kiloyears)	t_{PMI} (kiloyears)	t_{LMI} (kiloyears)	t_{CMI} (kiloyears)	t_{NMI} (kiloyears)	t_{2NMI} (kiloyears)
-429.9 **-429.9**	**-337.0** **-337.0**	**-242.8** **-239.3**	**-128.4** **-128.3**	**-8.240** **-7.714**	112	230
Chi Square:	3428 965	Data:	3272 1937	Parameters:	17	
Untuned Data Chi Square:	4917 1377					

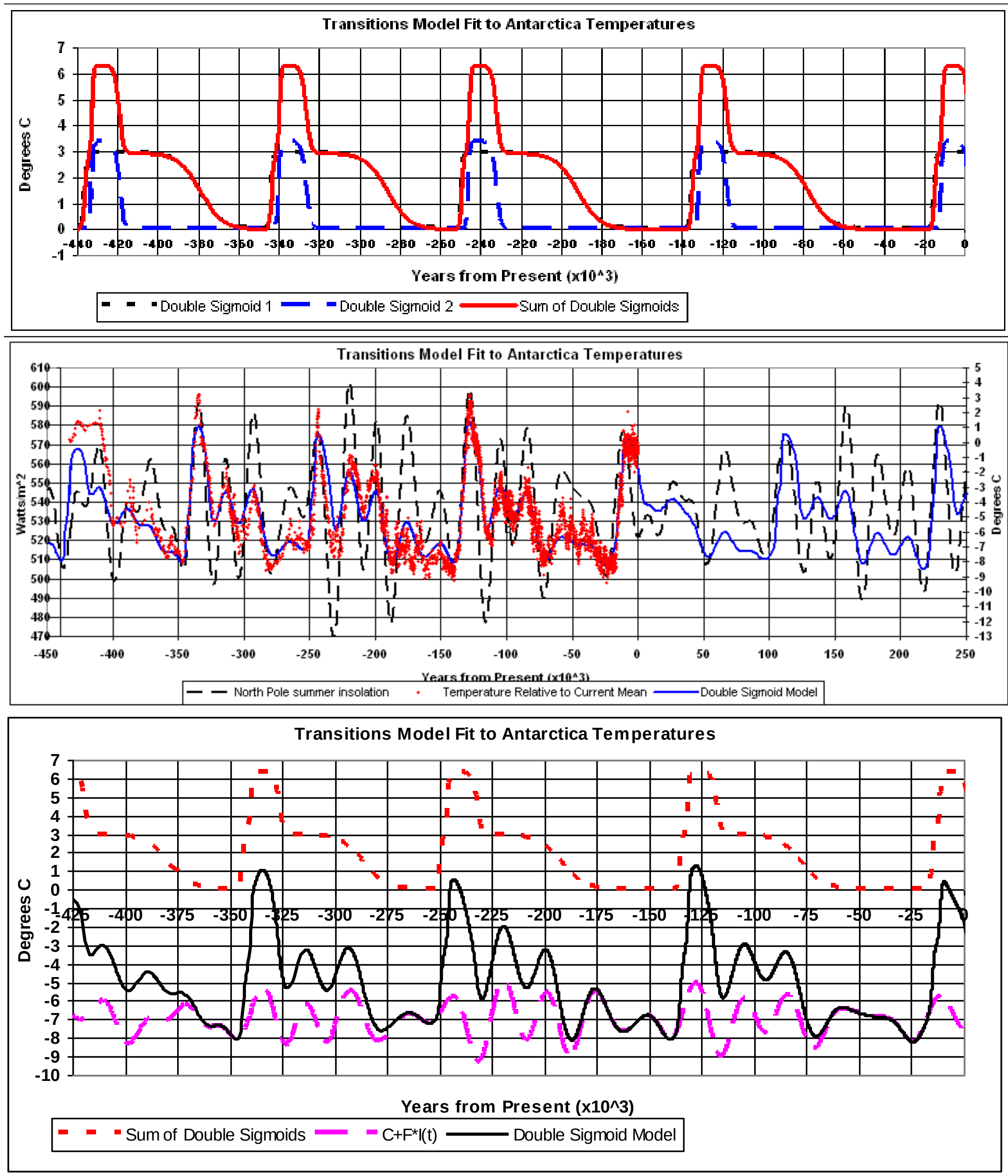


Figure 4a. The fit to the tuned Antarctica temperatures of the last four Major Ice Ages using the transitions model described above with two double sigmoids which have the same parameters for all seven Major Ice Ages. The pairs of double sigmoids for the Major Ice Ages are shown in the top graph. The components of the fit are shown in the bottom graph.

Note, for both double-sigmoids, the rapid onset of the first temperature-increasing transition between the glacial maxima and the Major Interglacials and the slow turnoff of one of them in the middle of the Major Ice Ages. However, the other one turns off rapidly at about the first insolation minima after the Major Interglacials. The short-term (about 13 kiloyears duration) one could be the far northern branch of the Great Ocean Conveyor Belt and the longer-term one (about 58 kiloyears) could be the more southerly northern part of the GOCB.

An interesting question is: How well can the data for the Last Major Ice Age be fitted without including the solar insolation? Figure 4b. shows such a fit. The fit is not very good (χ^2 =2270 compared to 965 for fit above with the solar insolation included); the solar insolation is very important to get a good fit.

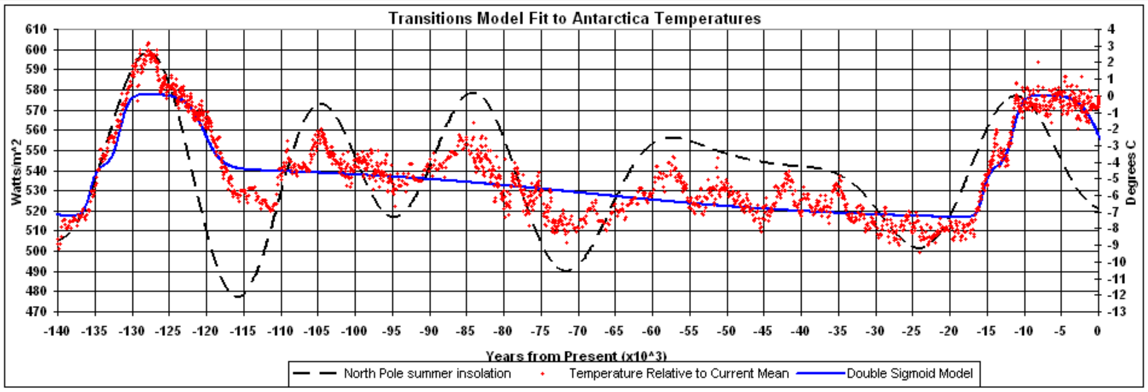


Figure 3c. A fit to the tuned Antarctica temperatures with only the two double-sigmoids in the model equation.

One could interpret the one-double-sigmoid fit of the last section as an approximation of the more accurate two-double-sigmoids fit of this section, which appears plausible in the comparison of the double-sigmoid transitions for the two fits in Figure 5.

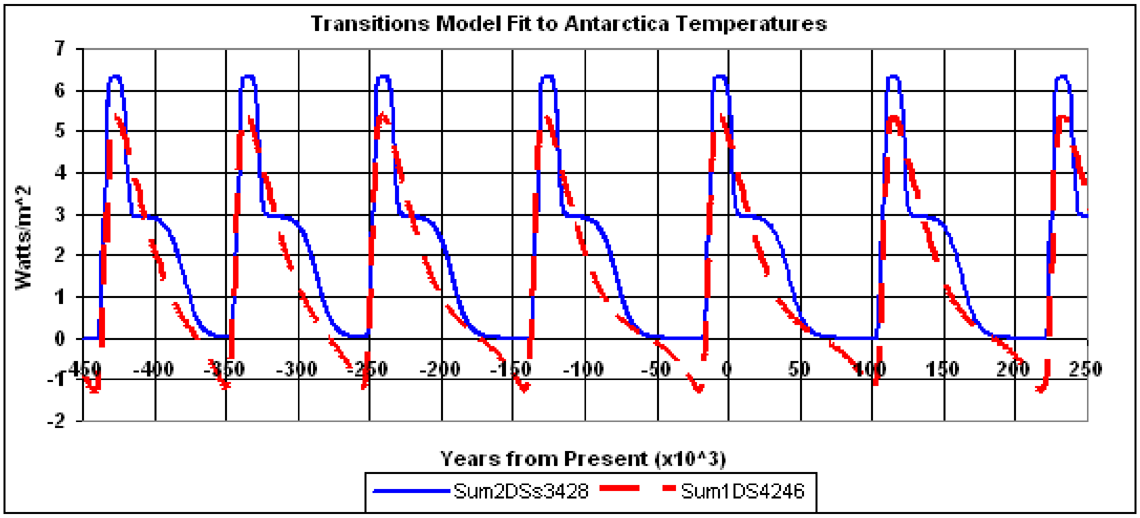


Figure 5. Comparison of the transition functions for the one-double-sigmoid fit and the two-double-sigmoid fit for the case of the same parameters for all Major Ice Ages.

The prediction for the two Next Ice Ages will be discussed in the conclusion.

Transitions Model's Fit with One Double Sigmoid with Different Parameters for each Major Ice Age

The result of the best fit obtained for the transitions model to the Antarctica atmospheric temperatures of the last four Major Ice Ages using one double sigmoid with different parameters for each Major Ice Age is shown in Table 5. and Figure 6. The two parameters for the later transition of the Next Major Ice Age double-sigmoid transitions are set equal to the corresponding parameters in Table 3. This appears to be a unique solution type; no other solution type was found that fit the data nearly as well as this solution. Some of the parameters have large uncertainties.

The same fits were made to the untuned data also, with a similar result, but, of course, a considerably higher chi squares.

In this version of the transitions model the t_j parameters were fixed at the insolation Major Interglacial values, since the transitions' positions c_{nij} were all varied and t_j only enters the equation as additive to the transitions' positions.

Note that χ^2 is less than the number of data minus the number of varied parameters, indicating that that errors on the measured temperature data are less than 1 C$^\circ$ or that the fit is over-determined.

Note that this case give a better fit to the data than the previous case (χ^2=2962 versus 3428 for the previous case).

Table 5. Fit parameters for the one-double-sigmoid transitions model with different parameters for each Major Ice Age (MIA). (**Red** indicates variable parameters.) (The j index in the equation is suppressed; it is represented by the different rows.) Note that the fit is to tuned data, with no allowance for a time delay between North-Pole insolation and Antarctica temperature. Therefore, the positions (c_{ij}) may have a time delay of a few thousand years. The last row in the table gives the chi square for a fit to the untuned data.

Double Sigmoid	s_1 (Degrees C)	c_{11} (kiloyears)	w_{11} (kiloyears)	c_{12} (kiloyears)	w_{12} (kiloyears)	F (Degrees C/(Watts/m^1))
DS1	**13.13**	**-21.56**	**19.02**	**5.127**	**22.91**	**0.03507**
DS2	**8.916**	**-7.243**	**3.993**	**7.367**	**21.13**	C (Degrees C)
DS3	**2.755**	**-2.414**	**0.4520**	**50.13**	**7.353**	**-25.04**
DS4	**11.27**	**-6.983**	**5.196**	**2.057**	**30.43**	
DS5	**10.16**	**-1.543**	**5.155**	13.03	30.73	
t_{3PMI} (kiloyears)	t_{2PMI} (kiloyears)	t_{PMI} (kiloyears)	t_{LMI} (kiloyears)	t_{CMI} (kiloyears)	t_{NMI} (kiloyears)	t_{2NMI} (kiloyears)
-415	-334	-244	-128	-11	112	230
Chi Square:	2962	Data:	3272	Parameters:	25	
Untuned Data Chi Square:	4939					

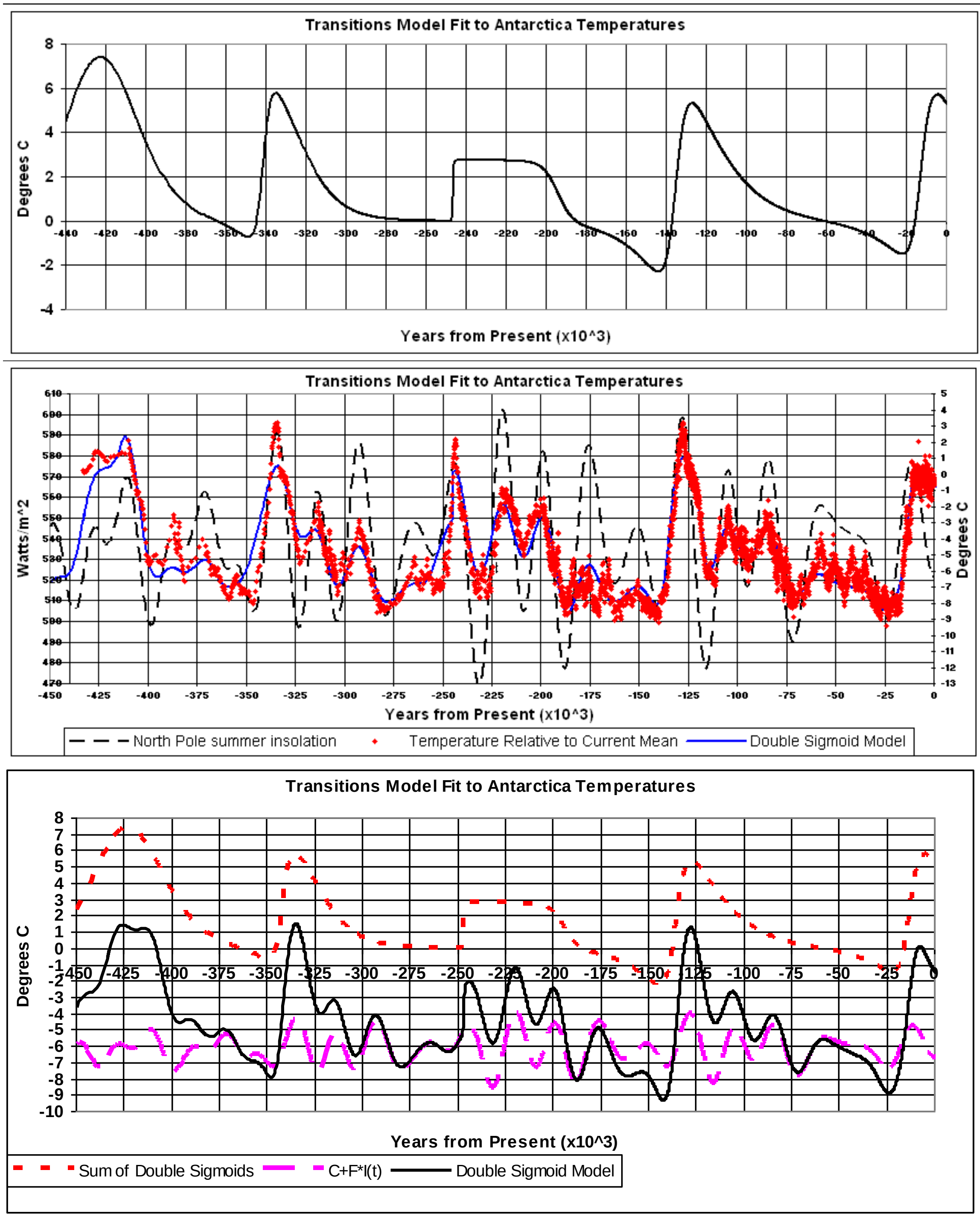


Figure 6. The fit to the tuned Antarctica temperatures of the last four Major Ice Ages using the transitions model described above with one double sigmoid with different parameters for each Major Ice Age. The double sigmoids for the Major Ice Ages are shown in the top graph. The components of the fit are shown in the bottom graph.

Figure 7. compares the transition functions for the one-double-sigmoid cases of constant parameters and different parameters for each Major Ice Age.

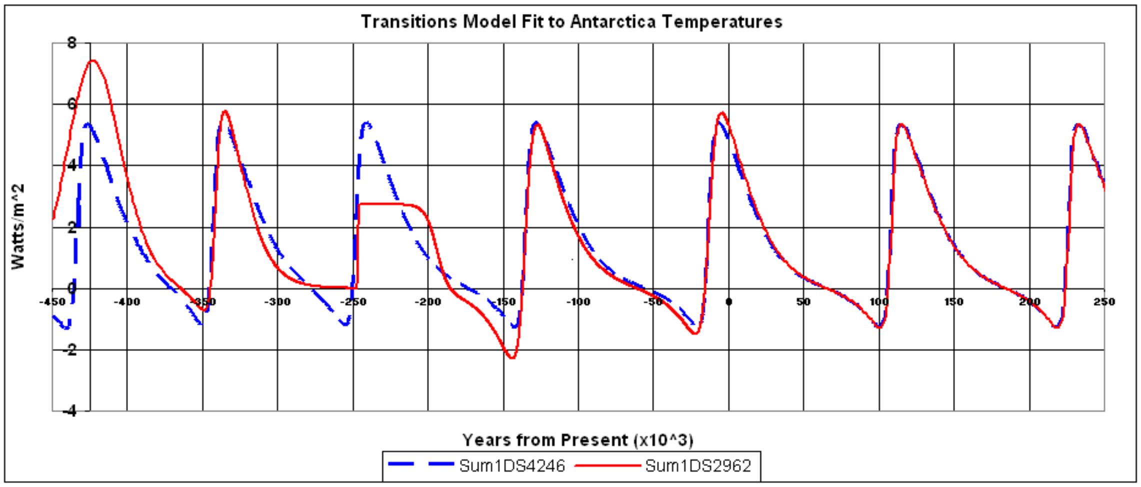


Figure 7. Comparison of the transition functions for the one-double-sigmoid cases of constant parameters and different parameters for each Major Ice Age.

Transitions Model's Fit with Two Double Sigmoids with Different Parameters for each Major Ice Age

There appear to be many possible solutions for this case. The solution given here is the simplest solution to interpret as turning on and off of Earth states.

The result of the best fit obtained for the transitions model to the Antarctica atmospheric temperatures of the last four Major Ice Ages using two double sigmoids with different parameters for each Major Ice Age is shown in Table 6. and Figure 7. The two parameters for the later transitions of the Next Major Ice Age double-sigmoid transitions are set equal to the corresponding parameters in Table 4.

In this version of the transitions model the t_j parameters were fixed at the insolation Major Interglacial values, since the transitions' positions c_{nij} were all varied and t_j only enters the equation as additive to the transitions' positions.

Note that χ^2 is less than the number of data minus the number of varied parameters, indicating that that errors on the measured temperature data are less than 1 C$^\circ$ or that the fit is over-determined.

Table 6. Fit parameters for two double sigmoids with different parameters for each Major Ice Age (MIA). (**Red** indicates variable parameters.) (The j index in the equation is suppressed; it is represented by the different rows.) Note that the fit is to tuned data, with no allowance for a time delay between North-Pole insolation and Antarctica temperature. Therefore, the positions (c_{ij}) may have a time delay of a few thousand years. The last row in the table gives the chi square for a fit to the untuned data.

Double Sigmoid	s_1 (Degrees C)	c_{11} (kiloyears)	w_{11} (kiloyears)	c_{12} (kiloyears)	w_{12} (kiloyears)	F (Degrees C/(Watts/m^1))
DS11	**7.079**	**-21.77**	**0.4340**	**21.14**	**90.48**	**0.03321**
DS12	**9.095**	**-7.932**	**6.445**	**5.760**	**33.56**	C (Degrees C)
DS13	**3.578**	**-0.8578**	**0.7576**	**56.37**	**7.747**	**-25.48**
DS14	**4.039**	**-8.199**	**2.907**	**55.63**	**50.92**	
DS15	**2.016**	**-4.323**	**0.6209**	50.88	10.84	
Double Sigmoid	s_2 (Degrees C)	c_{21} (kiloyears)	w_{21} (kiloyears)	c_{22} (kiloyears)	w_{22} (kiloyears)	
DS21	**4.008**	**-22.15**	**7.414**	**11.21**	**4.342**	
DS22	**15.54**	**-1.271**	**2.059**	**-0.4649**	**1.775**	
DS23	**26.67**	**-0.4987**	**2.532**	**0.3465**	**2.369**	
DS24	**3.516**	**-3.683**	**1.431**	**10.35**	**1.472**	
DS25	**4.991**	**-0.1037**	**2.931**	**22.84**	2.233	
t_{3PMI} (kiloyears)	t_{2PMI} (kiloyears)	t_{PMI} (kiloyears)	t_{LMI} (kiloyears)	t_{CMI} (kiloyears)	t_{NMI} (kiloyears)	t_{2NMI} (kiloyears)
-415	-334	-244	-128	-11	112	230
Chi Square:	1875	Data:	3272	Parameters:	49	
Untuned Chi Square:	3125	:				

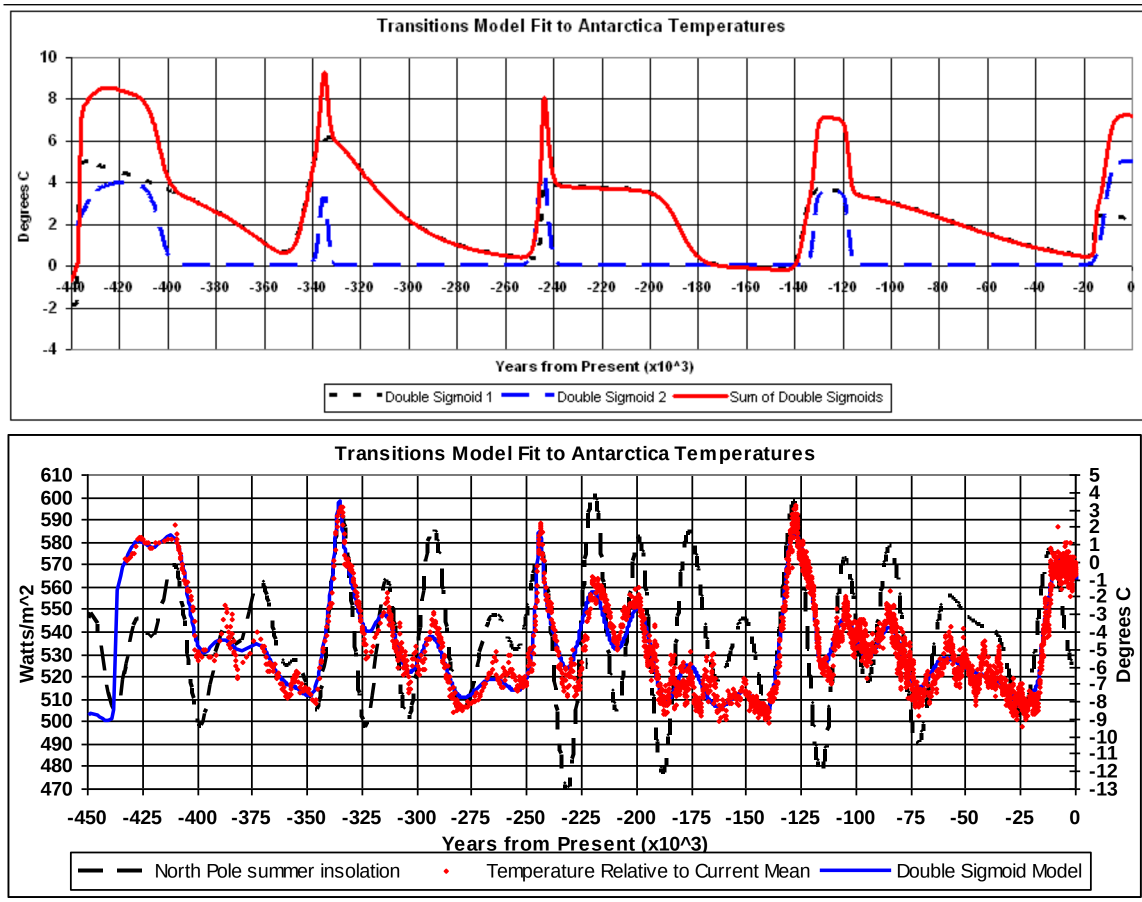


Figure 7. The fit to the Antarctica temperatures of the last four Major Ice Ages using the transitions model described above with two double sigmoids with different parameters for each Major Ice Age. The pairs of double sigmoids for the seven Major Ice Ages are shown in the top half of the graph.

One could interpret the one-double-sigmoid fit of the last section as an approximation of the more accurate two-double-sigmoids fit of this section, which appears plausible in the comparison of the double-sigmoid transitions for the two fits in Figure 8.

Also, one could interpret the two-double-sigmoids fit with the same parameters for all Major Ice Ages (Table 4. and Figure 4. above) as an approximation of the more accurate two-double-sigmoids fit of this section, which appears plausible in the comparison of the double-sigmoid transitions for the two fits in Figure 8.

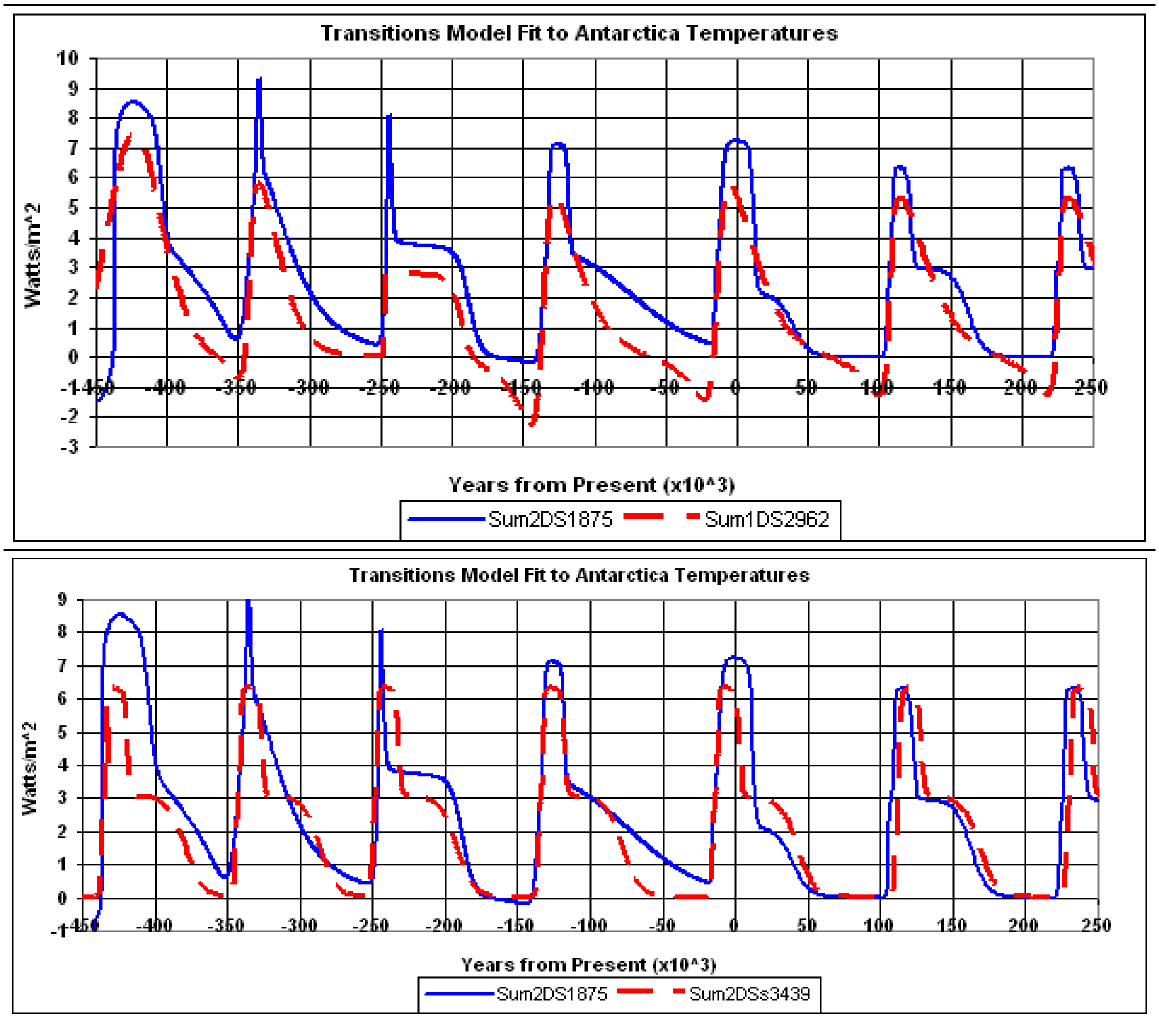


Figure 8. The top graph is the comparison of the transition functions for the one-double-sigmoid fit and the two-double-sigmoids fit for the case of different parameters for each Major Ice Age. The bottom graph is the comparison of the transition functions for the two-double-sigmoids fit with the same double-sigmoid parameters for all Major Ice Ages and the two-double-sigmoids fit for the case of different parameters for each Major Ice Age.

It is instructive to look more closely at the two double-sigmoid-transitions functions for this fit, as shown in Figure 9. Note that, near the -334 kiloyears, -128 kiloyears and -11 kiloyears insolation peaks, the initial transitions in the two double sigmoids are near to each other. (The +112 and +230 insolation peaks, of course, have no data to fit, so the transitions there are for the constant parameters two-double-sigmoid transitions model fit.) These initial transitions are close to the insolation maxima of the Major Interglacials.

The bottom half of Figure 9. shows the relative contributions of the summer North-Pole insolation and the two double sigmoids to the calculated temperatures. Note the closeness of the variation with time of the temperature to that of the insolation for the latter times of a Major Ice Age going into the glacial maxima. It appears that, without the two Earth states that are triggered on and off, the Earth, depending only on direct insolation, would be a cold place, indeed, oscillating between -6 ˚C and -10 ˚C below present temperatures. However, the Earth states get all or most of their energy from the insolation, so they must at some time release some of that energy into the atmosphere. The Earth states serve as solar-energy storage for periods of time, but periodically release some, probably most, of the stored energy into the atmosphere.

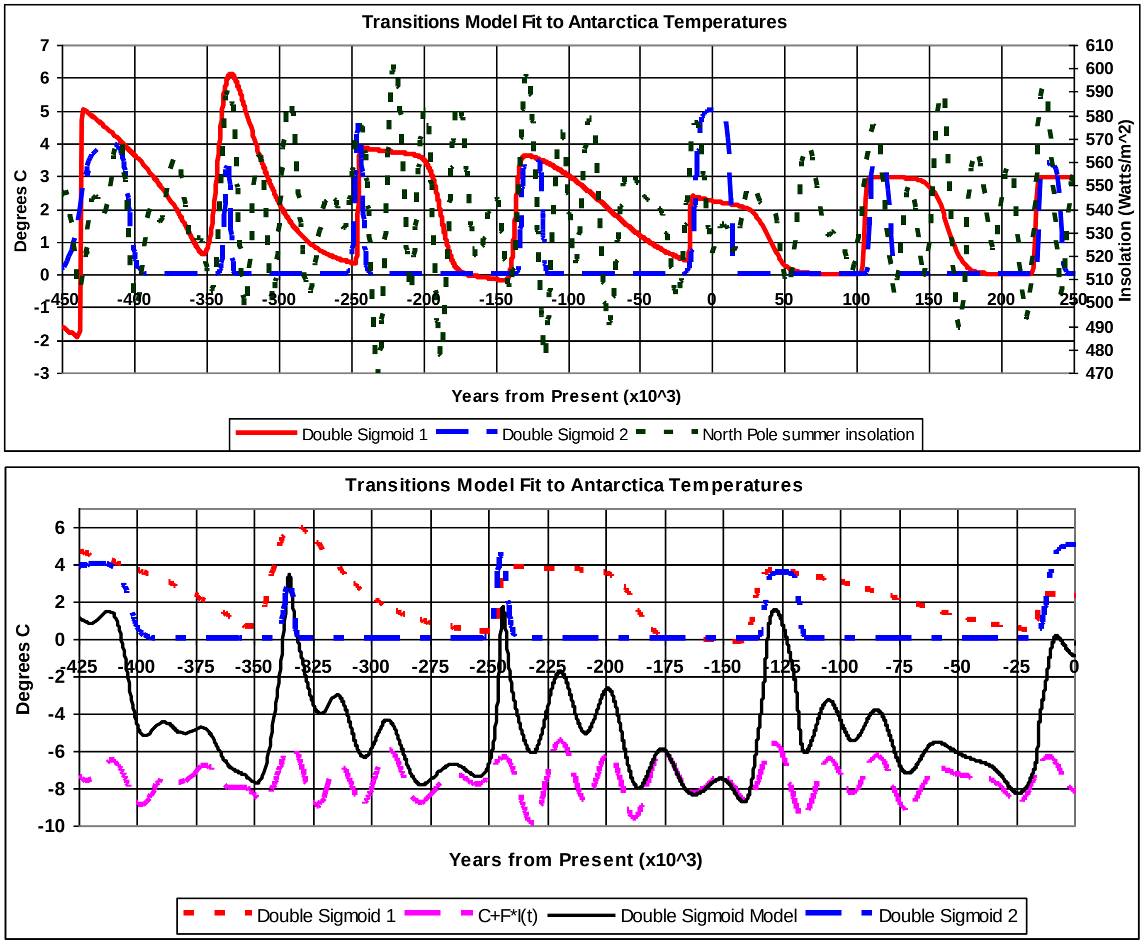
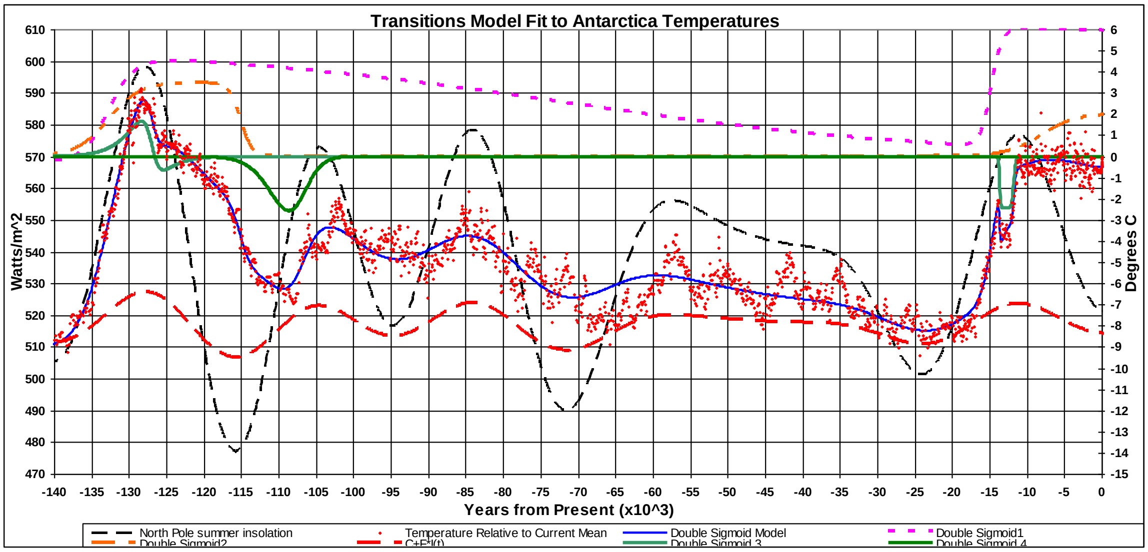


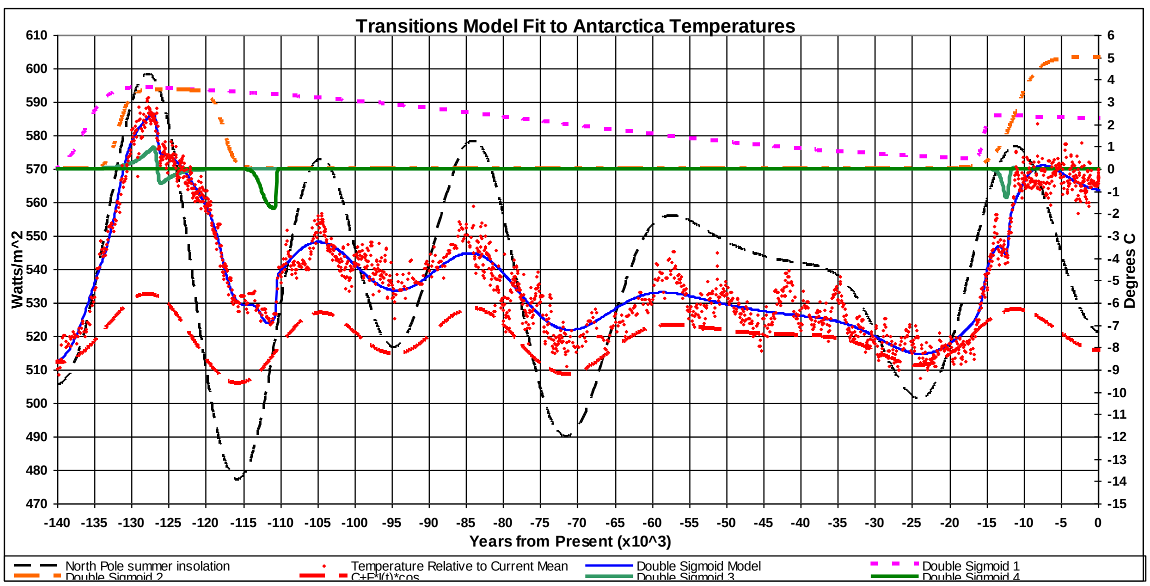
Figure 9. The top graph shows the two double-sigmoid-transitions functions (left axis) for the fit of Table 6. and the summer North-Pole insolation (right axis) The bottom graph shows the contributions of the two double sigmoids and the summer North-Pole insolation times the factor F plus the constant C to the calculated temperatures.

Possible Explanation of Fit in Terms of the Great Ocean Conveyor Belt

The fit given directly above for two double-sigmoids with different parameters for each Major Ice Age could be explained in terms of components of the Great Ocean Conveyor Belt (GOCB) as follows:

The fit to the Last Major Ice Age is as follows:





The top graph is for the fit to the original (untuned) data and the bottom graph is for the fit to the tuned data, a better fit.

Three other double sigmoids have been added to account for the temperature dips at about -126 kiloyears, about -112 kiloyears and about -13 kiloyears (the Younger Dryas), so that the first two double sigmoids at each of the two Major Interglacials can represent the remainder of the temperature beyond that caused by the North-Pole insolation.

The three major components of the fit are shown along with the fit and the North Pole insolation. The three components are:

1. North-Pole insolation times a factor F plus a constant C. Call it NPI.
2. Double-sigmoid 1 (DS1). It could be one component of the GOCB.
3. Double-sigmoid 2 (DS2). It could be another component of the GOCB

The time development of the Last Major Ice Age is as follows according to the fit:

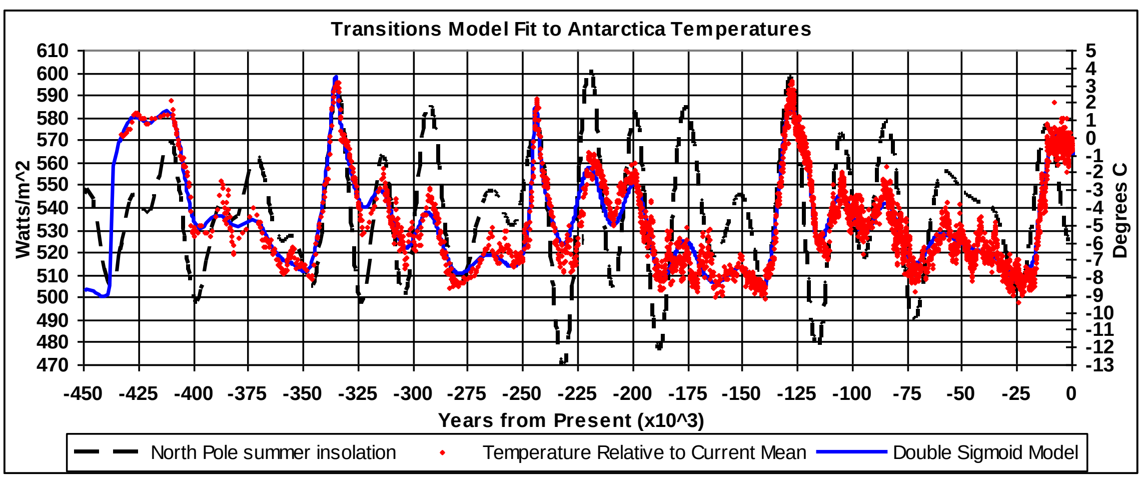
1. Start at about -140 kiloyears (at about the Last Glacial Maximum): DS1 begins its turn-on with **strength 4 degrees C**. The (hyperbolic tangent) **position ($c_{11} + t_{LMI}$) is about -136 kiloyears** and the **width (a measure of the turn-on rate) is about 3 kiloyears.**
2. About 5 kiloyears later DS2 begins its turn on with **strength 3.5 degrees C**, slightly less than the strength of DS1. The (hyperbolic tangent) **position ($c_{21} + t_{LMI}$) is about -131.5 kiloyears** and the **width (a measure of the turn-on rate) is about 1.5 kiloyears**, about half the width of DS1
3. Note the addition of NPI+DS1+DS2 to produce the Last Interglacial
4. At about -120 kiloyears DS2 begins to turn off with turn-off **position about -117.5 kiloyears** and **width 1.5 kiloyears**. It is essentially off by -115 kiloyears. From here on until the Last Glacial Maximum the temperature is determined by the sum NPI+DS1. Thus, DS2 has a lifetime of about 14 kiloyears, turn-on position to turn-off position, or about 15.5 kiloyears, adding in half of the sum of the two widths.
5. DS1 has a very slow turn-off with **position -72.5 kiloyears** and width **51 kiloyears.** It has fallen to less than 0.5 degrees C by the Last Glacial Maximum.
6. At about -17 kiloyears DS1 begins to turn on again, with **strength 2 degrees C**, at about half way up the North-Pole insolation rise into the Current Major Interglacial. (This is about half the strength it had for the Last Interglacial.) Its turn-on **position ($c_{11} + t_{CMI}$) is about -15.5 kiloyears** and its **width is 0.6 kiloyears**, about 5 times its turn-on rate for the Last Major Interglacial.
7. Also, at about -17 kiloyears DS2 begins to turn on again, with **strength 5 degrees C**, about 1.5 times more than for the Last Major Interglacial. Its **position ($c_{21} + t_{CMI}$) is about -11 kiloyears**, at the peak of the North-Pole insolation, and its **width is about 3 kiloyears**. (The relative strengths of DS1 and DS2 for the Current Major Interglacial have a large uncertainty because there are no data after 0 kiloyears to constrain them.)
8. Three “fine structure” events shown in the Antarctica temperature are fitted by extra double-sigmoids near times of -126 kiloyears, -111 kiloyears and -13 kiloyears (the Younger Dryas event). The fit to the first of these three events has an unusual structure of a rise followed by a dip. As explained in Appendix I, that structure is due to a turn on with a width larger than the time between the turn on and the turn off, the latter of which has a small width.

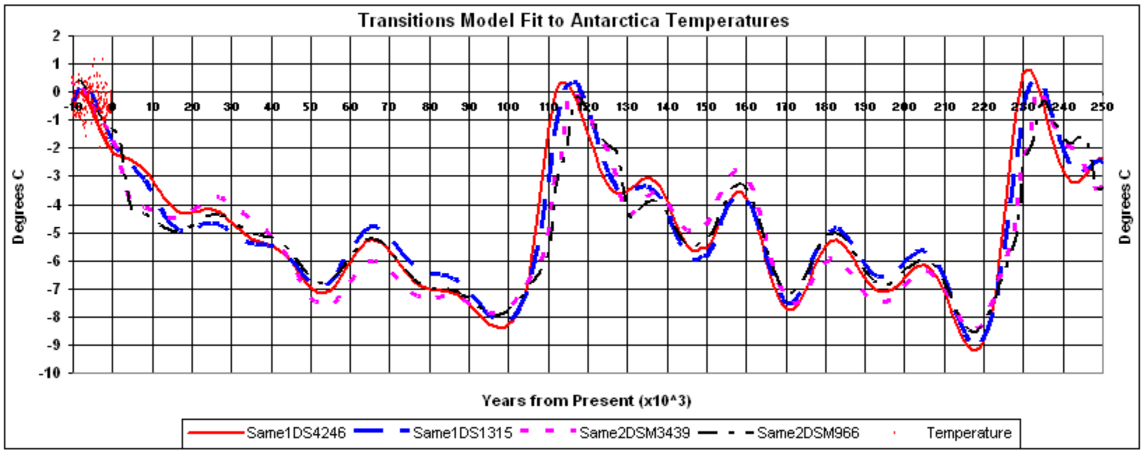
The time development described above does not depend on regarding the double sigmoids as Great Ocean Conveyor Belt transitions; they could be other Earth-state transitions. The following is a possible interpretation of the two double-sigmoids as two components of the GOCB, called G1=DS1 and G2=DS2:

1. Start in the Last Interglacial at about -128 kiloyears. Both components G1 and G2 are on. Melt water from the glacial ice causes G2 (probably the Northern North Atlantic component of the GOCB) to turn off over the period of -116 to -119 kiloyears. The melt water decreases the salinity at the location of the downwelling that drives G1, which stops the downwelling.
2. The G1 component, which starts off slightly stronger than G2, stays on but steadily declines over the course of the Last Major Ice Age. It declines because the solar heating of tropical waters declines over the Major Ice Age, thus decreasing the temperature difference between the incoming tropical waters and the cold Arctic waters at the Labrador downwelling. The salinity of the ocean has decreased because of reduced evaporation, which also works to turn off G2.
3. Since both G1 and G2 are essentially off by after the Last Glacial Maximum at about -18 kiloyears and the North Pole solar insolation is rapidly rising, the temperature differences and the salinity at the Labrador downwelling becomes large enough to turn G1 on again first, since it is further south, and then to turn on G2 again.
4. This cycle also occurs in the Major Ice Ages before the Last Major Ice Age.

Conclusion

The summer North-Pole solar insolation has been used with a double-sigmoid Earth-states transitions model for Major Ice Ages to fit the tuned and untuned Antarctica atmospheric temperatures, determined from deuterium concentrations in ice cores, for the last four Major Ice Ages. The fits predict Antarctica temperatures for the next two Major Ice Ages. Figure 10. shows a comparison of three of the fits and their predictions. The spread of the three predictions can be taken as a measure of the predictions temperature range for the next two Major Ice Ages.





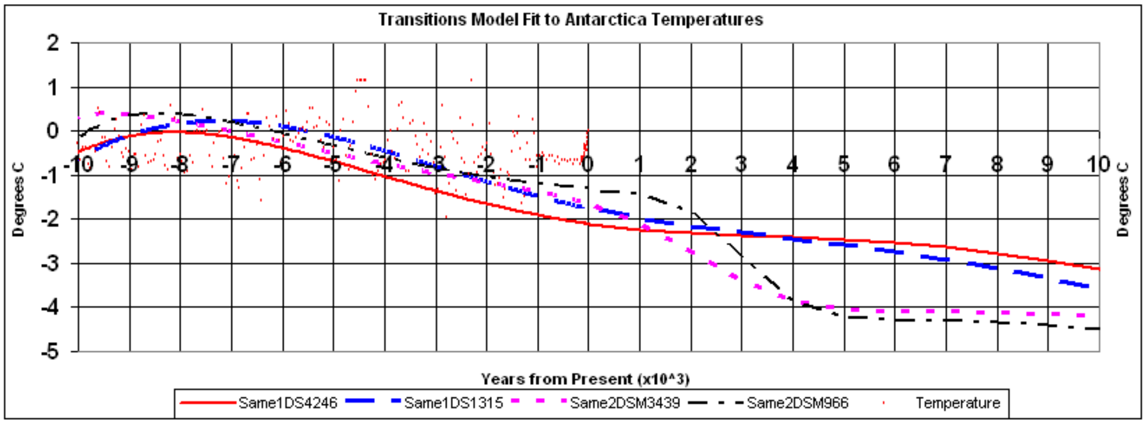


Figure 10. Comparison of the transitions-model's fits for the last four Major Ice Ages and predictions for the next two Major Ice Ages given in this paper. The bottom graph shows the last 10,000 years and the prediction for the last 1000 years and the next 10,000 years. (Note the rapid rise in temperature over the last few hundred years. The data for the last 1000 years were not used in the fits because of the influence of the use of fossil fuels by *Homo sapiens sapiens* on the temperature.)

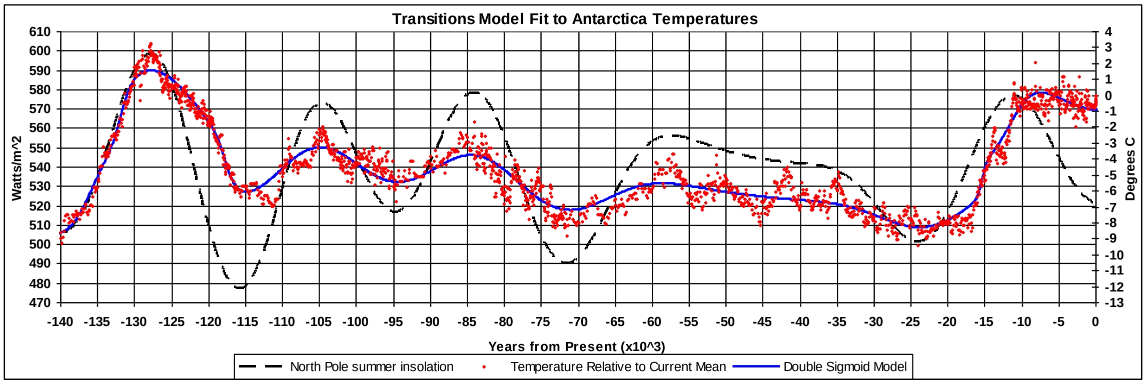
Note the relative lack of time variation of the Next Major Ice Age compared to the Last Major Ice Age, as noted in Table 1. The two-double-sigmoids fits predict a faster fall in temperatures from about 1000 to 6000 years from now than do the one-double-sigmoid fits.

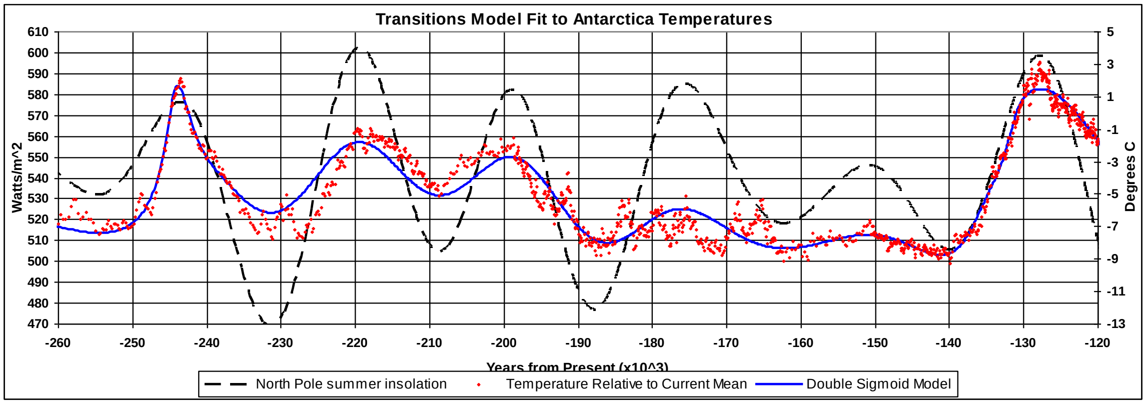
It appears that, after *Homo sapiens sapiens* (Hss) rapid use of all available fossil fuels over the next 1000 years, keeping the temperature warmer than it would be otherwise, Hss will be in for a great shock as the temperature rapidly falls to 4-5 C° lower than present at about 15-20 kiloyears from now. This will be followed by a slightly less rapid drop to about 7 C° lower than present at 50-55 kiloyears from now. Then there will be a rapid warm-up to about 5 C° lower than present at about 65 kiloyears from now. Then there will be a further drop to about 8 C° below present (the glacial maximum) at 95-100 kiloyears from now. Then there will be the usual rapid rise into the Next Major Interglacial with temperature about the same as now at about 115 kiloyears from now.

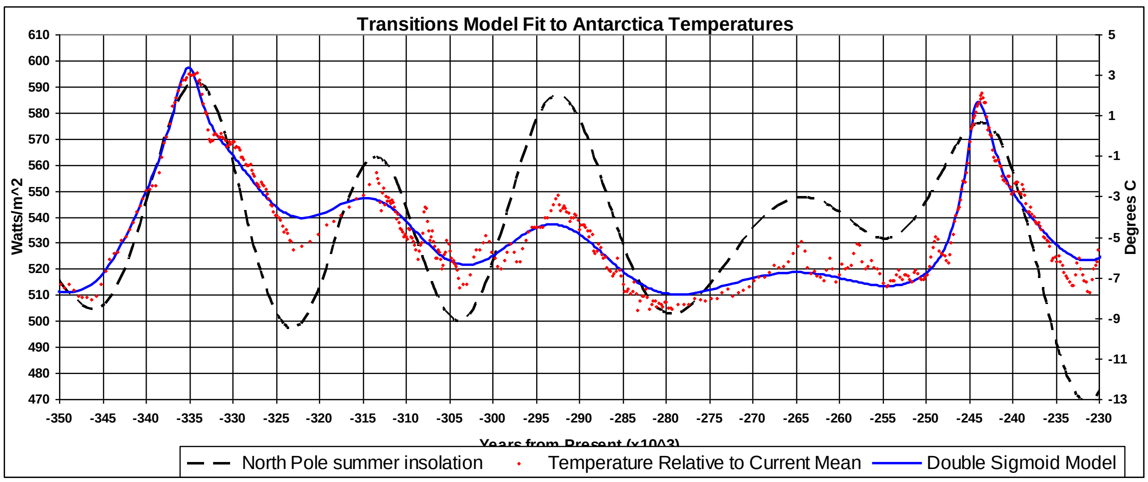
The temperatures of the Second Next Major Ice Age will be much more variable than the temperatures of the Next Major Ice Age, more similar to the Last Major Ice Age and the Penultimate Major Ice Age.

Some comments about global warming are appropriate here. It is certain that the Earth's climate is poised to enter the Next Major Ice Age. Perhaps global warming has delayed its commencement. However, the short-lived ocean current (or other Earth state) double-sigmoid can be made even shorter by the high temperatures of global warming, since fresh water put into the northern polar waters can trigger its cutoff. So global warming may trigger the cutoff sooner than predicted, causing the temperatures to plummet even faster than shown in the predictions given here. That is, global warming may trigger a fast entry into the Next Major Ice Age. The climate fine structure of *Homo sapiens sapiens* causing global warming will be similar to the fine structure of volcano eruptions or comet collisions.

Figure 11. shows the best fit of the two-double-sigmoid transitions model to the tuned Antarctica temperature data for the four past Major Ice Ags.


Transitions Model Fit to Antarctica Temperatures
Watts/m^2
Degrees C
Years from Present (x10^3)
North Pole summer insolation
Temperature Relative to Current Mean
Double Sigmoid Model


Transitions Model Fit to Antarctica Temperatures
Watts/m^2
Degrees C
Years from Present (x10^3)
North Pole summer insolation
Temperature Relative to Current Mean
Double Sigmoid Model


Transitions Model Fit to Antarctica Temperatures
Watts/m^2
Degrees C
North Pole summer insolation
Temperature Relative to Current Mean
Double Sigmoid Model

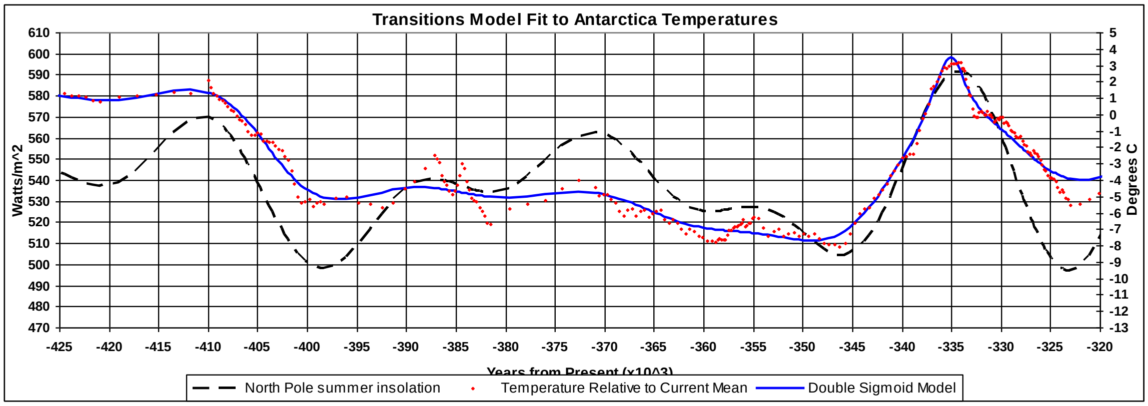


Figure 11. Fit of the two-double-sigmoid transitions model, with different parameters for each Major Ice Age, to the tuned Antarctica temperature data for the four past Major Ice Ages.

The results of the transitions-model's fits to the tuned Antarctica temperature data can be interpreted as freezing of water and otherwise releasing energy from the Earth into the atmosphere at insolation minima and melting of ice and otherwise storing energy in the Earth at insolation maxima over a Major Ice Age and by turning on of parts or all of the Great Ocean Conveyor Belt quickly in the vicinity of a Major Interglacial insolation peak, with the GOCB's turning off occurring at a latter insolation minimum, possibly by parts.

One can calculate residual Antarctica temperatures by subtracting off the best fit of the transitions model from the tuned Antarctica measured temperatures, and thus obtain a crude measure of other climate driving forces on atmospheric temperature besides insolation and the Earth-states transitions modeled by the double sigmoids. The residual temperatures for the two-double-sigmoids model of Table 6. and Figure 7. are shown in Figure 12. The author can supply tables of the residual temperatures.

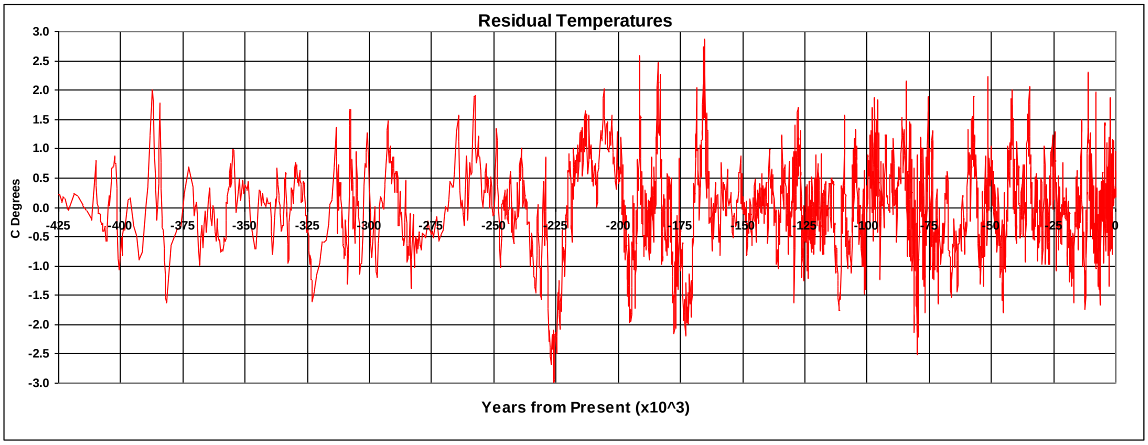


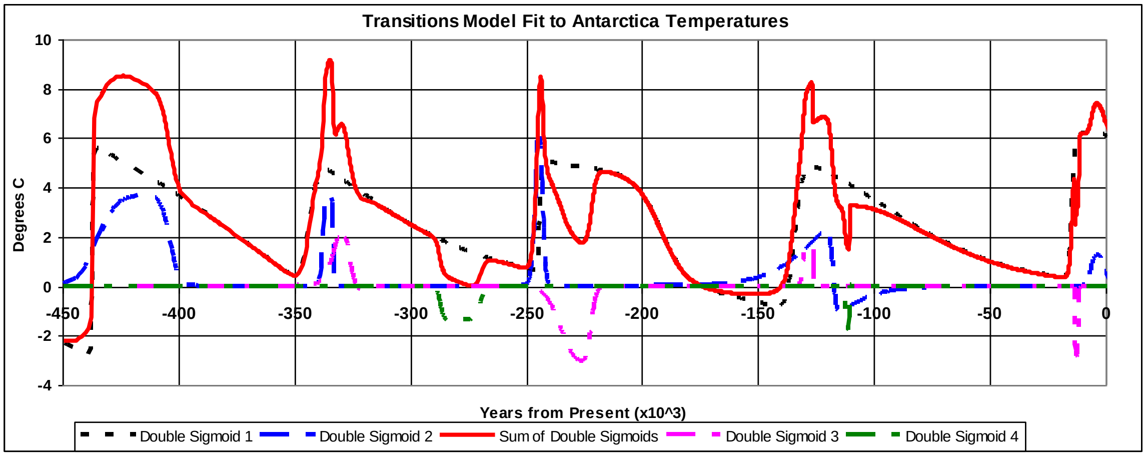
Figure 12. Residual temperatures obtained by subtracting the two-double-sigmoid transitions model calculation of temperatures from the measured Antarctica temperatures57.

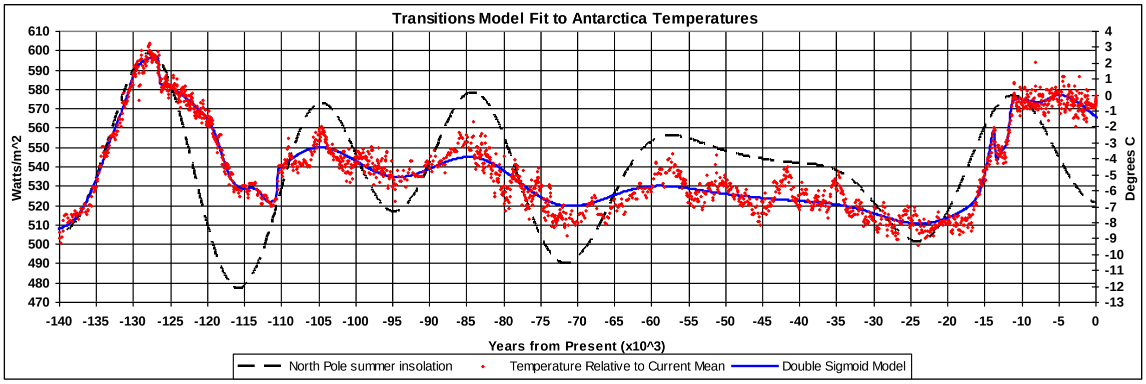
Part II. Major Ice Ages Fine Structure

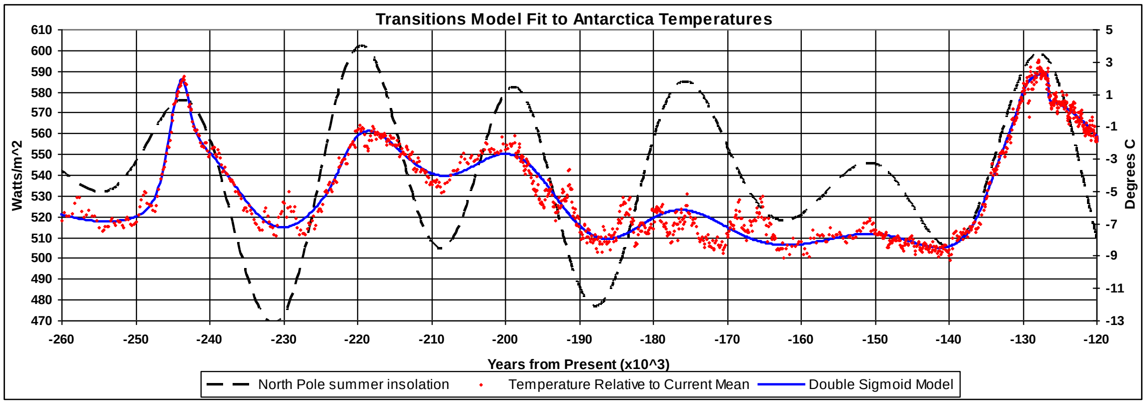
After completing the work described above a successful attempt was made to add more transition functions to represent the “kinks” or “fine structure” seen in the last four Major Ice Ages. Table II-1. contains the fit parameters and Figure II-1. shows one of the fits. This fit accounts for the Younger Dryas (a temperature dip at about -12.5 kiloyears[3]) going into the Current Major Interglacial and other fine structure in the other four past Major Interglacials. It also accounts for the fast rise at about -107 kiloyears.

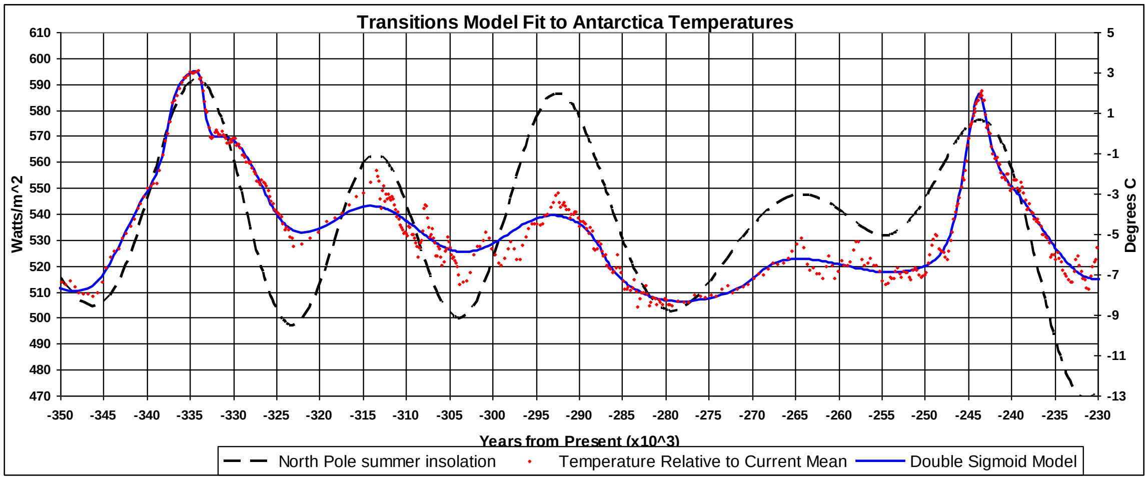
Table II-1. Fit parameters for four double sigmoids with different parameters for each Major Ice Age (MIA). (**Red** indicates variable parameters.) (The j index in the equation is suppressed; it is represented by the different rows.) Note that the fit is to tuned data, with no allowance for a time delay between North-Pole insolation and Antarctica temperature. Therefore, the positions (c_{ij}) may have a time delay of a few thousand years. A similar, but less good fit $\left(\chi^2 = 1716\right)$ was obtained with the untuned data.

Double Sigmoid	s_1 (Degrees C)	c_{11} (kiloyears)	w_{11} (kiloyears)	c_{12} (kiloyears)	w_{12} (kiloyears)	F (Degrees C/(Watts/m[1]))
DS11	**8.519**	**-21.77**	**0.4182**	**12.42**	**88.88**	**0.02757**
DS12	**5.459**	**-9.192**	**3.579**	**9.910**	**63.48**	C (Degrees C)
DS13	**4.474**	**-0.04464**	**0.7059**	**52.19**	**11.00**	**-22.33**
DS14	**6.358**	**-6.040**	**3.402**	**33.04**	**53.55**	
DS15	**5.929**	**-3.315**	**1.257**	50.88	10.84	
Double Sigmoid	s_2 (Degrees C)	c_{21} (kiloyears)	w_{21} (kiloyears)	c_{22} (kiloyears)	w_{22} (kiloyears)	
DS21	**3.690**	**-20.07**	**7.947**	**11.12**	**3.575**	
DS22	**3.744**	**-3.536**	**1.014**	**0.6849**	**0.5545**	
DS23	**27.78**	**-0.5547**	**1.823**	**0.1872**	**1.589**	
DS24	**3.913**	**3,421**	**21.76**	**10.09**	**1.494**	
DS25	**1.417**	**4.870**	**1.457**	10.35	2.233	
Double Sigmoid	s_3 (Degrees C)	c_{31} (kiloyears)	w_{31} (kiloyears)	c_{32} (kiloyears)	w_{32} (kiloyears)	
DS32	**-20.63**	**6.679**	**4.119**	**5.873**	**4.962**	
DS33	**-3.271**	**8.500**	**6.445**	**22.26**	**2.145**	
DS34	**1.676**	**-2.467**	**0.9014**	**1.506**	**0.1022**	
DS35	**-2.838**	**-2.539**	**0.1404**	**-0.5803**	**0.2668**	
Double Sigmoid	s_4 (Degrees C)	c_{41} (kiloyears)	w_{41} (kiloyears)	c_{42} (kiloyears)	w_{42} (kiloyears)	
DS42	**-1.380**	**46.75**	**1.525**	**64.62**	**2.380**	
DS44	**-1.810**	**15.44**	**0.9289**	**17.51**	**0.1137**	
t_{3PMI} (kiloyears)	t_{2PMI} (kiloyears)	t_{PMI} (kiloyears)	t_{LMI} (kiloyears)	t_{CMI} (kiloyears)	t_{NMI} (kiloyears)	t_{2NMI} (kiloyears)
-415	-334	-244	-128	-11	112	230
Chi Square:	1398	Data:	3272	Parameters:	78	


Transitions Model Fit to Antarctica Temperatures
Degrees C
Years from Present (x10^3)
Double Sigmoid 1
Double Sigmoid 2
Sum of Double Sigmoids
Double Sigmoid 3
Double Sigmoid 4


Transitions Model Fit to Antarctica Temperatures
Watts/m^2
Degrees C
Years from Present (x10^3)
North Pole summer insolation
Temperature Relative to Current Mean
Double Sigmoid Model


Transitions Model Fit to Antarctica Temperatures
Watts/m^2
Degrees C
Years from Present (x10^3)
North Pole summer insolation
Temperature Relative to Current Mean
Double Sigmoid Model


Transitions Model Fit to Antarctica Temperatures
Watts/m^2
Degrees C
Years from Present (x10^3)
North Pole summer insolation
Temperature Relative to Current Mean
Double Sigmoid Model

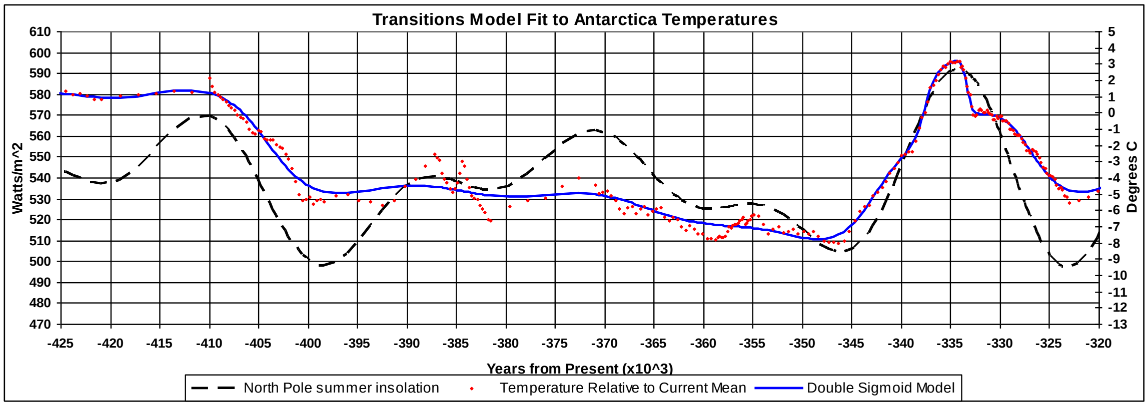


Figure II-1. A fit to the four past Major Ice Ages using double sigmoids to account for fine structure, especially near the Major Interglacials.

The fit result for the time region of the Younger Dryas event up to present time is shown in Figure II-2.

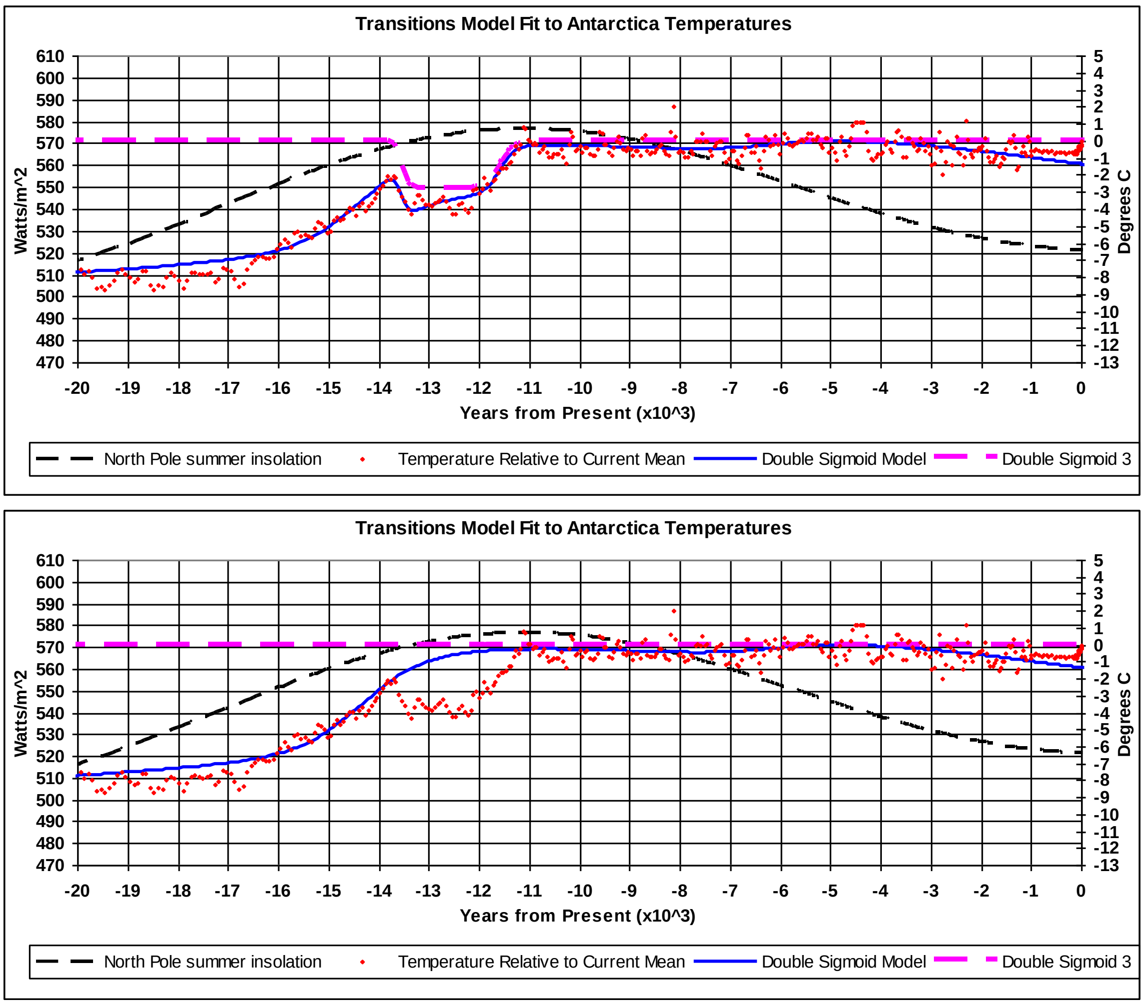


Figure II-2. The fit to the Antarctica tuned temperatures in the region of the Younger Dryas event. The dotted curve is the North-Pole insolation. The dashed curve is the double sigmoid to produce the Younger-Dryas event. The bottom graph shows the fit without the Younger-Dryas double sigmoid.

Figure II-3. shows how the insolation and the sum of the double sigmoids contribute to the calculated temperatures.

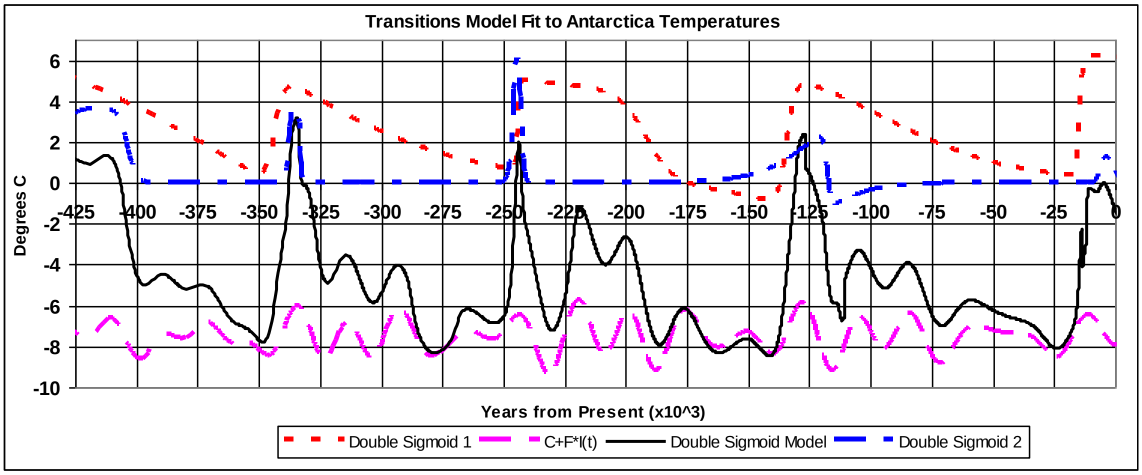


Figure II-3. The contributions of the largest two double sigmoids and the summer North-Pole insolation times the factor F plus the constant C to the calculated temperatures.

Figure II-4. shows the smaller double sigmoids besides the two largest ones. For a discussion of the unusual shapes of most of these double sigmoids see Appendix I; it is because the two transition positions (c_1, c_2) are closer together than the values of the widths (w_1, w_2). That is, the second transition of the event begins before the first is finished.

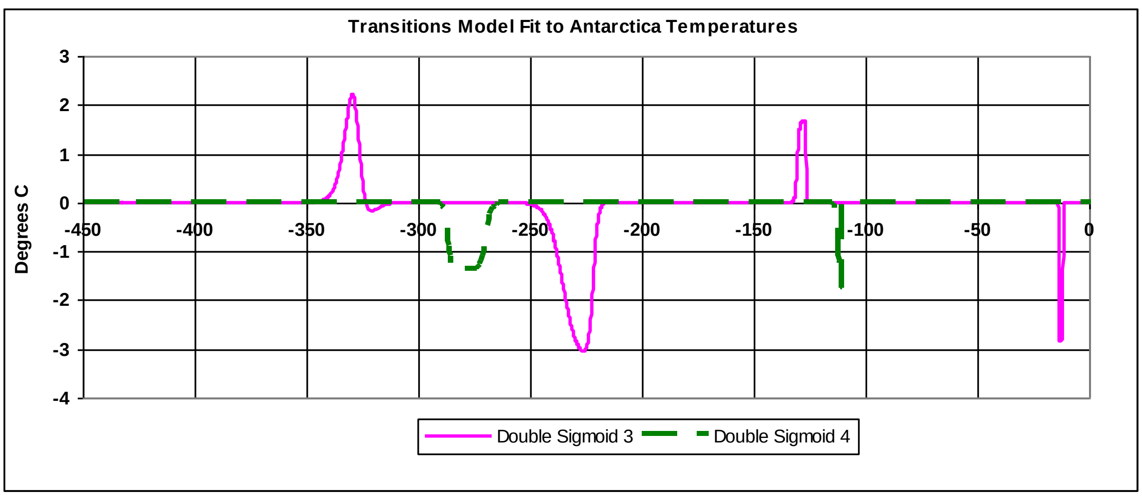


Figure II-4. The double sigmoids minus the two largest ones. The solid curve on the right is for is the Younger-Dryas event.

The residual temperatures for this best of all solutions are shown in Figure. II-5. It is seen that the major events have been accounted for by fitting double sigmoids to them in the fit above.

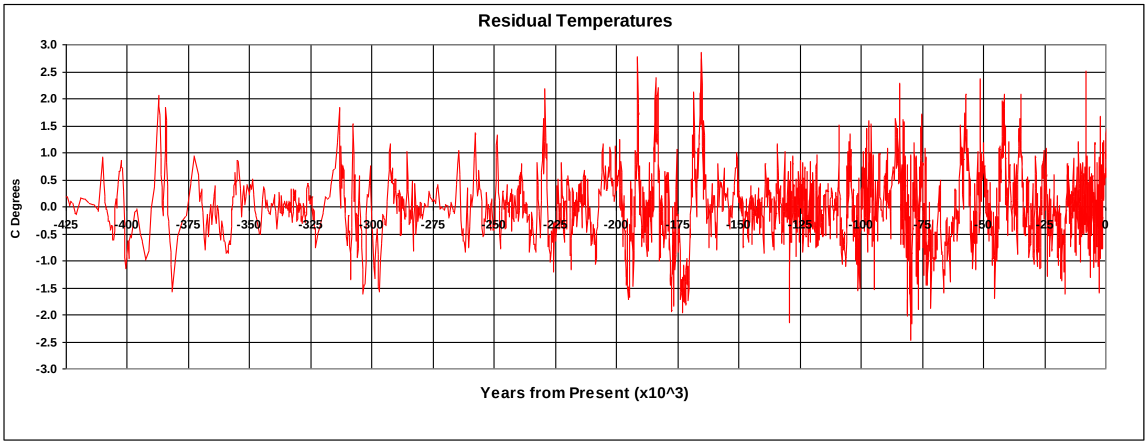


Figure II-4. The residual temperatures calculated for the best fit solution of Table. 7. Compare this graph to the graph in Figure 12.

Solar Irradiance Variability

In Figure II-1. one can see that there appear to be some short-period periodicity in the Antarctica temperatures that has not been accounted for in the fits so far, especially for the Last Major Ice Age. A cosine term was added directly or was put in as a modulation of the solar irradiance; the latter gave the best fit. It was also found that a second cosine term improved the fit, but no further terms did. Figure II-5 shows the fit to the last four Major Ice Age whose parameters are given in Table II-2.

The equation for the model is modified by replacing the $F \cdot I(t)$ term by the term

$F \cdot I(t)\left[1 + A_1 \cos\left(\frac{2\pi}{T_1}t + P_1\right) + A_2 \cos\left(\frac{2\pi}{T_2}t + P_2\right)\right]$. The cosine terms modulate the effect of the solar insolation on the temperature. This can be interpreted as a time modulation of the solar irradiance, which heretofore in this paper has been considered to be a constant.

The problem for the future is to explain why the solar irradiance has two periodic modulation terms with periods of about 7 and 48 kiloyears.

Table II-2. Fit parameters for four double sigmoids with different parameters for each Major Ice Age (MIA). (**Red** indicates variable parameters.) (The j index in the equation is suppressed; it is represented by the different rows.) Note that the fit is to tuned data, with no allowance for a time delay between North-Pole insolation and Antarctica temperature. Therefore, the positions (c_{ij}) may have a time delay of a few thousand years. A similar, but less good fit $\left(\chi^2 = 1716\right)$ was obtained with the untuned data.

Double Sigmoid	s_1 (Degrees C)	c_{11} (kiloyears)	w_{11} (kiloyears)	c_{12} (kiloyears)	w_{12} (kiloyears)	F (Degrees C/(Watts/m[1]))
DS11	**7.884**	**-21.77**	**0.4181**	**13.65**	**91.28**	**0.02496**
DS12	**5.339**	**-8.656**	**3.786**	**13.236**	**63.28**	C (Degrees C)
DS13	**4.701**	**0.2131**	**0.6239**	**49.52**	**10.19**	**-20.88**
DS14	**4.803**	**-7.337**	**2.432**	**41.21**	**53.13**	
DS15	**6.040**	**-3.324**	**1.419**	50.88	10.84	
Double Sigmoid	s_2 (Degrees C)	c_{21} (kiloyears)	w_{21} (kiloyears)	c_{22} (kiloyears)	w_{22} (kiloyears)	
DS21	**4.282**	**-17.71**	**9.613**	**11.37**	**3.537**	
DS22	**3.363**	**-3.696**	**0.8921**	**0.6172**	**0.5135**	
DS23	**27.93**	**-0.1927**	**1.896**	**0.4369**	**1.390**	
DS24	**3.396**	**-0.5078**	**3.621**	**9.737**	**1.386**	
DS25	**0.9995**	**4.959**	**0.3595**	10.35	2.233	
Double Sigmoid	s_3 (Degrees C)	c_{31} (kiloyears)	w_{31} (kiloyears)	c_{32} (kiloyears)	w_{32} (kiloyears)	
DS32	**-20.52**	**6.145**	**4.052**	**5.463**	**4.791**	
DS33	**-3.271**	**8.500**	**6.445**	**22.26**	**2.145**	
DS34	**2.561**	**-3.064**	**0.9560**	**1.340**	**0.4770**	
DS35	**-2.618**	**-2.540**	**0.1177**	**-0.6213**	**0.2616**	
Double Sigmoid	s_4 (Degrees C)	c_{41} (kiloyears)	w_{41} (kiloyears)	c_{42} (kiloyears)	w_{42} (kiloyears)	
DS42	**-1.774**	**46.23**	**1.299**	**63.58**	**2.882**	
DS44	**-2.182**	**15.29**	**1.175**	**17.50**	**0.1516**	
Cosine Parameters	A_1 (unitless)	T_1 (kiloyears)	P_1 (radians)	A_2 (unitless)	T_2 (kiloyears)	P_2 (radians)
	0.01311	**6.959**	**6.551**	**0.02203**	**48.25**	**5.817**
t_{3PMI} (kiloyears)	t_{2PMI} (kiloyears)	t_{PMI} (kiloyears)	t_{LMI} (kiloyears)	t_{CMI} (kiloyears)	t_{NMI} (kiloyears)	t_{2NMI} (kiloyears)
-415	-334	-244	-128	-11	112	230
Chi Square:	1302	Data:	3272	Parameters:	78	

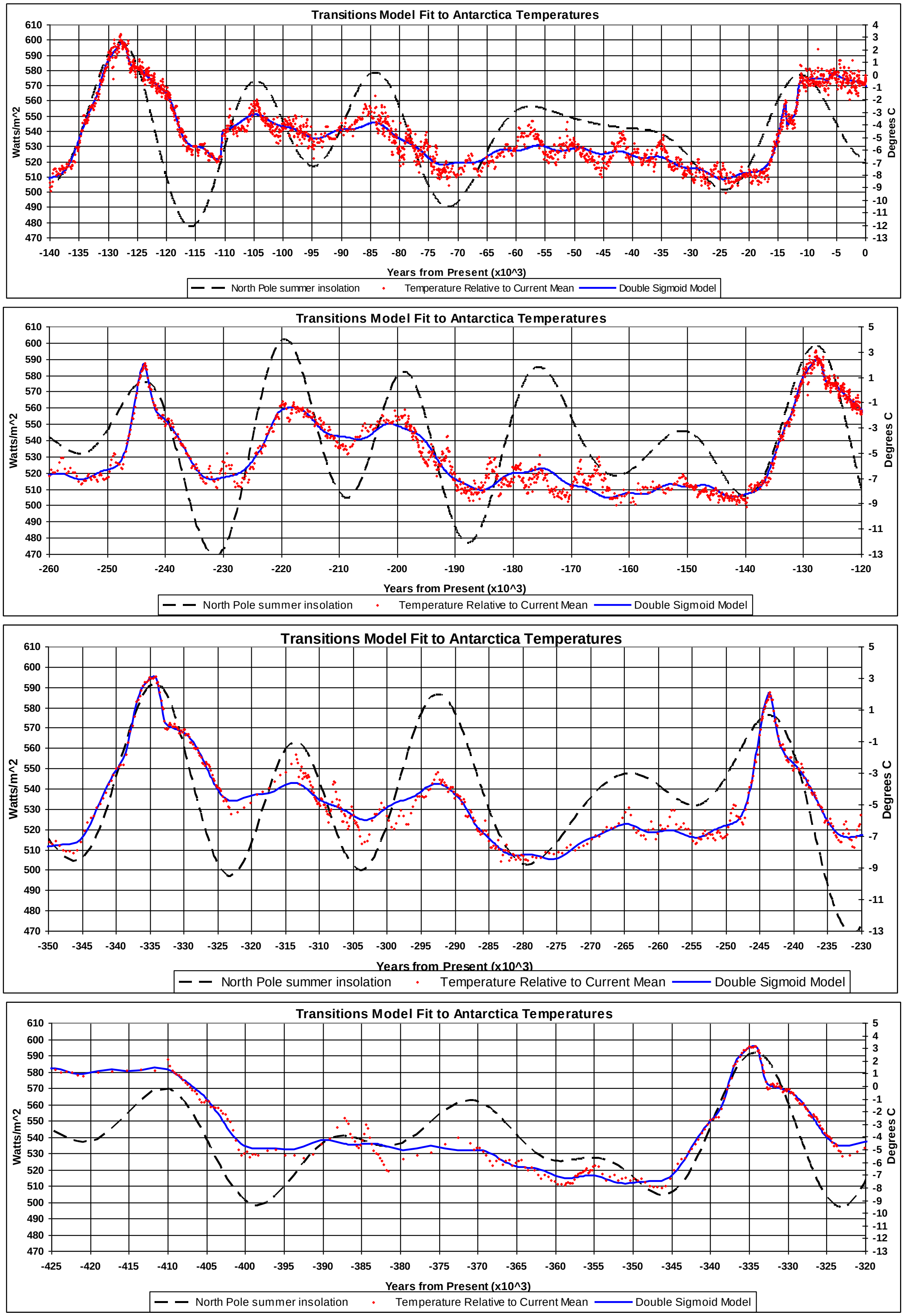


Figure II-5. Fit to the Last Major Ice Age by the two-double-sigmoids transitions model with two cosine waves modulating the solar irradiance. Note the wiggles in the fit compared to Figure II-1.

More About Global Warming

The fit with fine structure in the previous section yields the results for -20 kiloyears to +20 kiloyears shown in Figure II-6.

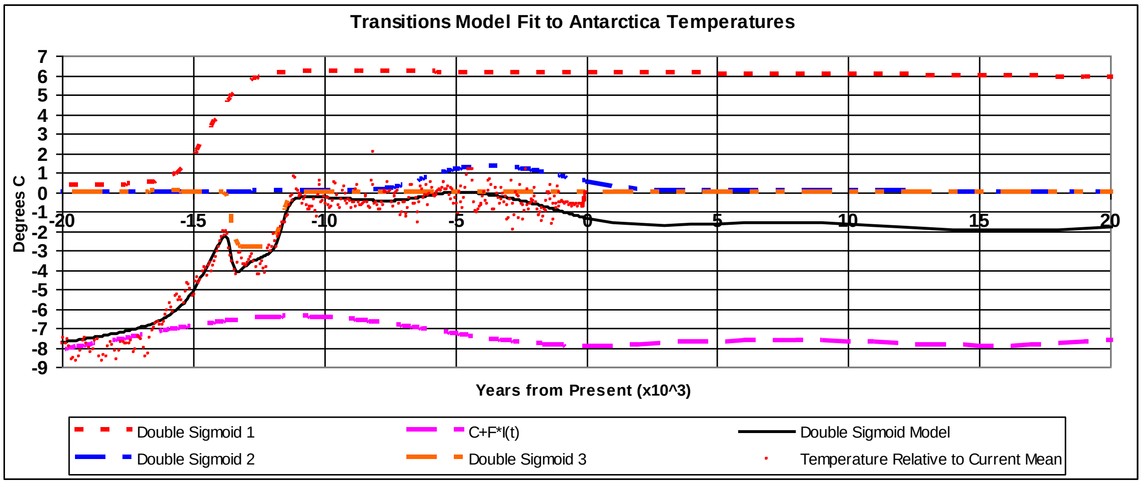


Figure II-6. The three double sigmoids in the vicinity of the Current Interglacial for the fit in the previous section.

For the two-double-sigmoids fit with the same parameters for all Major Ice Ages (Table 4. and Figure 4.) the -20 kiloyears to +20 kiloyears region is as shown in Figure II-7.

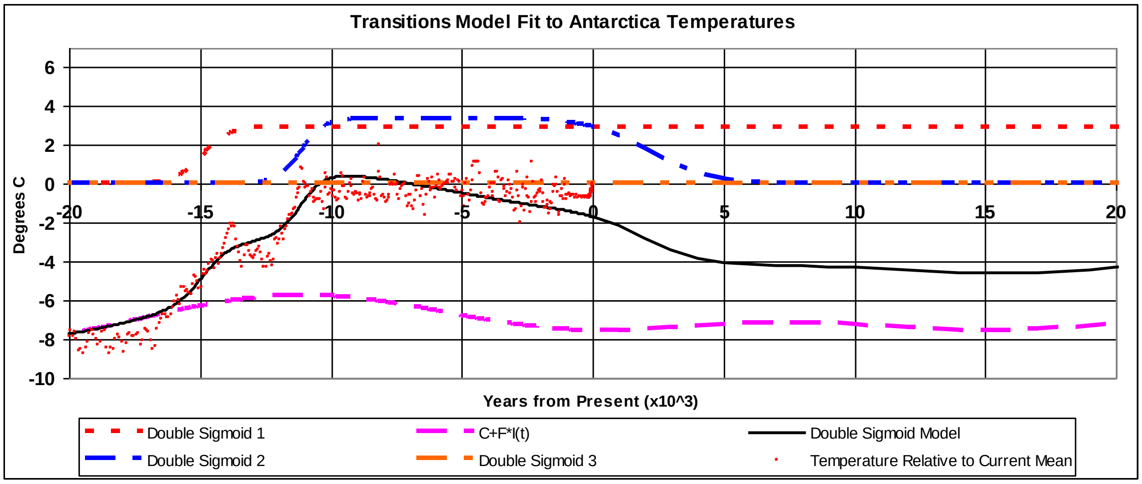


Figure II-7. The two double sigmoids in the vicinity of the Current Interglacial for the two-double-sigmoids fit with the same parameters for all Major Ice Ages.

The two figures above give a range of what can be expected within the next 20,000 years. One of the Earth states will turn off sometime between 3,000 and 5,000 years from now.

If one of the Earth states is the Great Ocean Conveyor Belt, extreme Global Warming could cause the turnoff to occur sooner and/or faster, rather than 3,000 or more years from now, which the two graphs indicate would cause a rapid temperature drop of 1.5 to 3.5 C °. If both of the Earth states are different parts of the GOCB, then Global Warming could cause both to turn off sooner with rapid temperature changes up to 6 C °, which, of course, would be disastrous for humans.

One might argue that humans could use the remaining oil, gas and coal in the earth to heat up the earth with its energy and CO_2 emissions greenhouse blanket for an indefinite period of time to make up for the turnoff of the GOCB. That would require a tremendous amount of heat creation and trapping, because the direct solar contribution to the climate is small compared to the GOCB contribution. (See Figure II-3 above.) It also would require a fast ramping up of energy use to counteract the fast turnoff of the GOCB. Anyway, an easy calculation using the world oil/gas/coal reserves, multiplied by a reasonable factor of choice, and the present rate of energy-use increase per year shows that getting energy from those fuels will peak in about 1000 years. If humans decide that it is alright to use radioactive fuels, that might be extended another 1000 years or longer, if population decreases and/or energy use per capita decreases. (However, nuclear fuel would mostly add heat with little CO_2 emissions to trap heat.) In any case humans will not be able to affect the climate after a thousand years or so, as they are inadvertently doing now.

It is expected, as shown in Figure II-8, that the Earth's temperature rise in recent years follows the increase in the Earth's population.

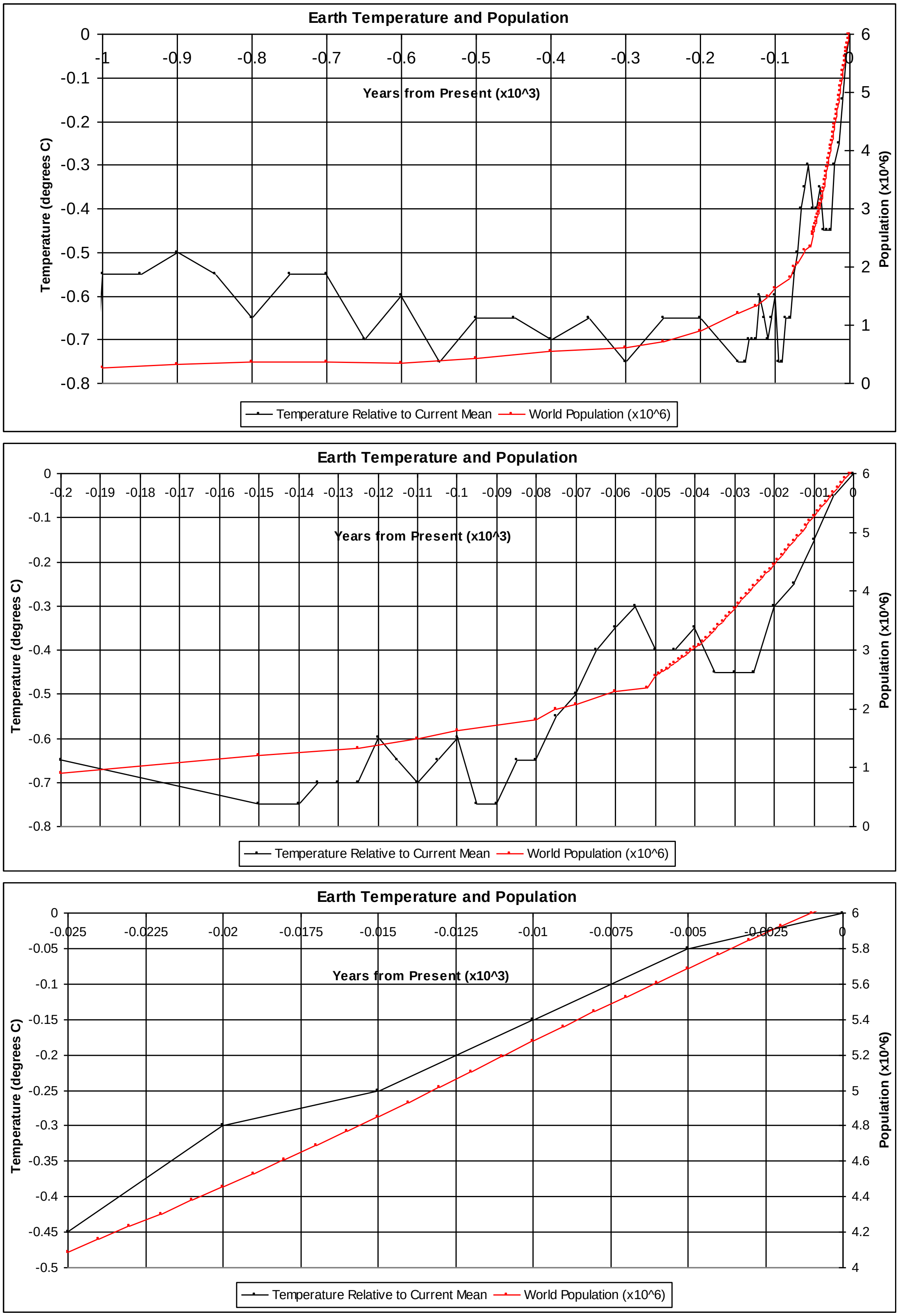


Figure II-8. Earth temperature and population in the near past.

One can calculate the ratio of temperature to population for the last 25 years and fit a parabola to it to predict the temperature for the next 50 years by using the projected population, as shown in Figure II-9. It is seen that, if the population grows as projected, the temperature will rise by about 0.5 degrees C in 50 years. Compare this to the drop in temperature by 0.2 degrees C in the last Little Ice Age (c1350-c1850), as shown in Figure II-8. Such a temperature rise by itself will cause drastic changes in the climate, but the danger of such a temperature rise turning off some or all of the Great Ocean Conveyor (GOCB) Belt is much greater. Turning off the GOCB will cause temperatures to drop by several degrees C, perhaps ushering in the next 100,000-year Major Ice Age sooner than it might otherwise occur.

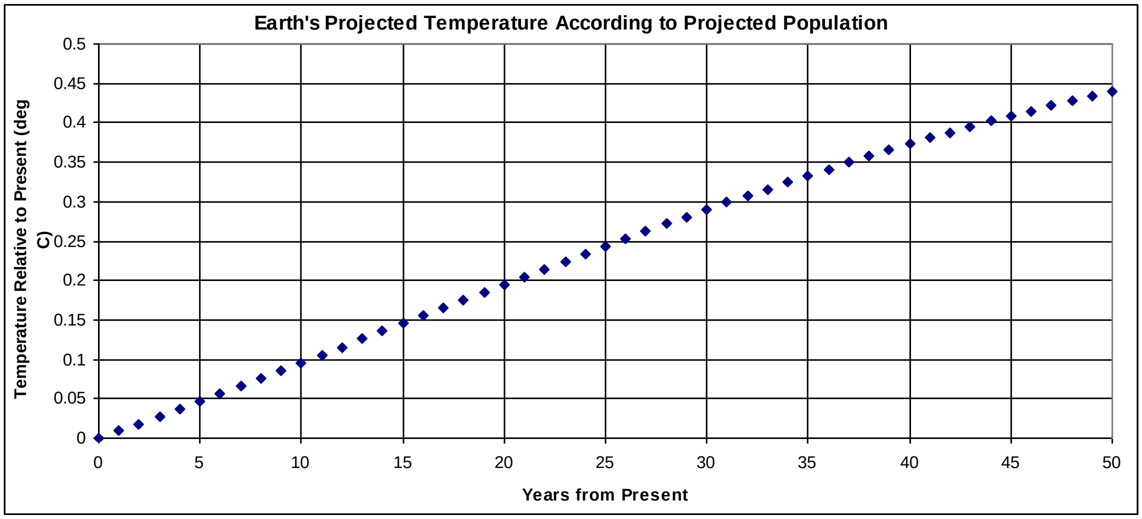


Figure II-9. Earth temperature projected into the future by means of projected population. The equation for the projection is $T = P\left(-0.01105t^2 + 1.5209t - 0.0459\right)/1000$, where P=population in units of 10^6 , t= time relative to the year 2000 in years, and T= temperature in degrees C relative to the year 2000.

Evolution has apparently been the fastest when climate has drastically changed, due to ice ages, comet collisions, etc. It appears certain that human evolution will accelerate at various points over the Next Major Ice Age.

Letter to the editor of The Roanoke Times about global warming:

Dear Editor:

Given the recent release of the movie "The Day After Tomorrow" about global warming and the near total disregard of the President and the Congress of the United States of the dangers of global warming, it appears to be a good time for a rational informed discussion of the dangers.

The movie has some good science in it about global warming, especially the parts about the possibility of global warming turning off the North Atlantic ocean current and the huge change in climate that would cause. However, the time scale for the disasters global warming could bring is probably much too small. The danger is probably at time scale of 100 years, or perhaps decades, instead of months or weeks as in the movie.

Over the last 2.5 million years, since the Isthmus of Panama was created by the collision of South America with North America shutting off the connection between the Pacific and Atlantic oceans, the Earth has been in ice ages of about 100,000 years duration for most of the time. Between the mostly cold times there have been "brief" respites (called interglacials), of about 5,000 years duration, of about 8 degrees Celsius (multiply by 9/5 to get about 14.5 degrees Fahrenheit) warmer than the coldest time (called the glacial maximum), which was always just before the warm interglacial. The present interglacial has been very unusual in that it has lasted about 10,000 years. (This extra long "respite" allowed civilization to develop.) (Humans left Africa 80-50,000 years ago during a particularly cold time to populate the rest of the Earth, because cold temperatures turn most of Africa into a desert.) (The last glacial maximum was about 18,000 years ago.)

The cold times vary over a wide range of temperatures of about 3 to 8 degrees Celsius colder than now, which variation is caused by the variation in the solar energy that strikes the upper atmosphere above the North Pole due to the actions of the other planets and the moon on the orbit and tilt of the Earth. (The solar energy at the North Pole is the important forcing factor because that is where most of the land is on which glaciers can form.) The interglacials are caused by the turning on of the Great Ocean Conveyor Belt (GOCB) that carries the water heated by the sun in the tropics to the North and South pole areas of the Earth. Usually after about 5,000 years the interglacials end, because the GOCB turns off, and the Earth plunges back into a long ice age for about 100,000 years.

Ocean surface water in the tropics is heated by the Sun. This hot water gathers a small amount of salt from the runoff of the continents as it moves north and south near the ocean surface, which gets highly concentrated in the surface water as some of the water evaporates to produce the rain on the continents, which then causes more runoff. When the salty surface water reaches the north and south polar areas it is greatly cooled by the very cold water there. Salty water is more dense than pure water and cold water is more dense than hot water. So, when the conditions are just right in the polar regions the salty and cold surface water rapidly sinks to near the bottom of the ocean north of Iceland and near Labrador and near Antarctica. That cold water then travels near the bottom of the ocean, losing salt along the way, back to the tropics to rise to the surface to be heated again in a cycle that lasts several thousand years. This Great Ocean Conveyor Belt is a heat pump moving heat from the tropics to the polar regions. That is what makes eastern North America and Europe warm and what melts the glaciers during the interglacials; without the GOCB the Earth would be several degrees Celsius colder than it is now.

So, in the long 10,000-year interglacial in which the Earth is now probably near the end, we should worry about the possibility of global warming turning off the Great Ocean Conveyor Belt sooner than it would "naturally" turn off. This could happen because the warmer temperature due to global warming melts more of the polar ice than would otherwise melt. This melt water is salt free and is warmer than the polar water usually is; so, when it mixes with the salty water coming from the tropics, it reduces the salinity of the water and keeps the water from getting as cold as usual, which may stop the sinking of the water in the polar regions and cause the GOCB to turn off sooner than it would otherwise.

It is possible that global warming over the last several thousand years, as human population on the Earth was growing rapidly and, thus, human activity was releasing carbon dioxide into the atmosphere creating the greenhouse effect, has kept the next ice age from beginning. But the rapid increase in temperature over the last 100 years of about 0.6 degrees Celsius is a completely different ball game. (That increase is largely due to the rapid increase in the use of coal and petroleum for energy, which releases carbon dioxide into the atmosphere causing the greenhouse effect.) The increase is closely related to the increase in the Earth's population and population is projected to continue growing at a high rate. I calculate that, given the projected population increase, the temperature will rise about 0.5 degrees Celsius over the next 50 years. (Other calculations give a much higher temperature increase.) There is a very real danger that this magnitude of temperature increase will trigger the turnoff of the Great Ocean Conveyor Belt sooner and sooner and more rapidly than normal for the past 2.5 millions years. In the past the turnoff intervals have been of the order of a few hundred to thousands of years. With global warming the turnoff time could be more like 100 years, or possibly even decades. The more rapid the turnoff the more likely the occurrence of super storms as depicted in the movie "The Day After Tomorrow." The increase in violent storms in recent years is due to global warming.

The description above is simplified from the more complicated actual situation. There appear to be at least two components of the GOCB that can turn on and off separately. The component that is neglected in the discussion above has a much longer time interval between its rapid turn on going into an interglacial and its slow turn off, about 50,000 years or longer, and is stronger. If global warming were enough to turn off, also, this second component soon and rapidly, humans would be in much more danger over the next thousand years, perhaps even to the point of extinction. Another complication is the fact that higher temperature increases the amount of carbon dioxide that is released from the ocean and land and the amount of water vapor put into the atmosphere, which provides a positive greenhouse feedback to increase the temperature even more. Yet another complication is that a component of the GOCB has turned off for "brief" periods in the past; for example, about 13,000 years ago as the Earth was rising fast into the present interglacial, part of the GOCB turned off for about 1,500 years to lower temperatures about 3 degrees Celsius. (This event is called the "Younger Dryas", named after a flower.) For those who want more details about the actual situation, see my web page http://www.roperld.com/Science/TransitionsModelMIA.pdf .

The discussion above explains how global warming, counter intuitively, could trigger the next ice age much sooner than it would otherwise occur.

What can be done about this “imminent”” global-warming danger? It is easy to say, but difficult to do. Reducing Earth's population growth would be most helpful, but the current U.S. administration is working against world population control. Reducing the per capita use of coal and petroleum for energy would also be helpful, but the current U.S. administration is working against that, also. The use of coal and petroleum **will** automatically be reduced in the period of about 1000 years into the future at the current rate of usage, because of the finite amount that is available. (World population probably also will be reduced automatically in unpleasant ways.) It would be wise to reduce coal and petroleum use for frivolous energy now and use it instead to prepare for the next ice age, which will occur in the next thousands of years, by building the necessary non-carbon energy infrastructure.

Finally, there is the nagging question as to whether humans are intelligent and future-oriented enough to realize the dangers and do something about them, looking into the future hundreds and thousands of years, or even decades. There are those claim hold that future technology will save humans from the dangers of global warming and the next ice age. My imagination is not bold enough to believe that nuclear energy will keep the next ice age from happening, since ice ages are the "normal" situation over time.

Yours, L. David Roper

Many Transitions Fit to Last Major Ice Age

After the success of fitting some of the Little Ice Ages with double sigmoids given above, the natural next question is: Can one fit almost all of the Little Ice Ages during the Last Major Ice Age with double sigmoids? The answer is “Yes,” as shown in Table II-3 and Figure II-10 below.

Note the two temperature-increasing transitions that occurred 4 to 5 kiloyears apart for the Last Interglacial and 6 to 7 kiloyears apart for the Current Interglacial. Up to the present time both of those states are in effect; the first of them was greatly modulated by a double sigmoid that represents the Younger Dryas event. For the Last Interglacial the second one (that turned on at -130.6 kiloyears) turned off at -117.7 kiloyears; whereas, the first one turned off very slowly over the entire course of the Last Major Ice Age.

That long turn off was modulated by 13 double sigmoids. One of these centered at 168 kiloyears was a very broad one at a deep insolation minimum, which itself can be thought of as modulated by 3 or 4 of the other 12 double sigmoids. 12 of these 13 double sigmoids are turn offs followed by turn ons.

Table II-3. Fit parameters for the Last Major Ice (including the rise into it, but excluding the last 1000 years): using 18 double sigmoids

Double Sigmoid	s	c1	w1	c2	w2	Comments	F
DS1	3.885	-134.94	2.106	-65.13	52.68	Long-term DS for Last Interglacial	0.04143
DS2	3.077	-130.61	1.449	-117.72	1.209	Short-term DS for Last Interglacial	C
DS3	-1.464	-126.51	0.05651	-123.00	2.927	Dip after peak of Last Interglacial	-29.07
DS4	-12.65	-111.14	1.395	-110.73	1.134	Sharp dip at first minimum after Last Interglacial	
DS5	-1.730	-108.61	0.4738	-105.57	0.5374	Square dip	
DS6	-1.351	-103.91	0.4680	-100.01	0.3387	Square dip	
DS7	-3.693	-79.20	2.074	-79.05	0.3716	Dip-rise	
DS8	1.616	-76.57	1.19E-08	-72.23	6.647	Dip-rise	
DS9	1.452	-68.97	0.9193	-67.43	0.02173	Rise at minimum at about -67 kiloyears	
DS10	-46.13	-63.01	15.35	-61.94	14.18	Broad dip from -60 to -80 kiloyears	
DS11	-13.94	-55.70	1.696	-55.54	2.450	Rise-dip	
DS12	-57.65	-44.77	2.909	-44.58	2.741	Deep dip at about -46 kiloyears	
DS13	-1.278	-40.79	1.86E-06	-35.88	0.1177	Square dip	No. of data
DS14	-1.094	-34.19	0.2150	-26.64	0.02544	Square dip	1377
DS15	-3.330	-17.93	6.368	-15.88	0.8503	Large dip-rise	No. of parameters
DS16	4.139	-13.88	0.4074	39.64	11.98	Long-term DS for Current Interglacial	92
DS17	-3.496	-13.60	0.1376	-11.64	0.3663	Younger-Dryas dip	Chi Square
DS18	2.225	-6.596	2.046	11.95	1.643	Short-term DS for Current Interglacial	414

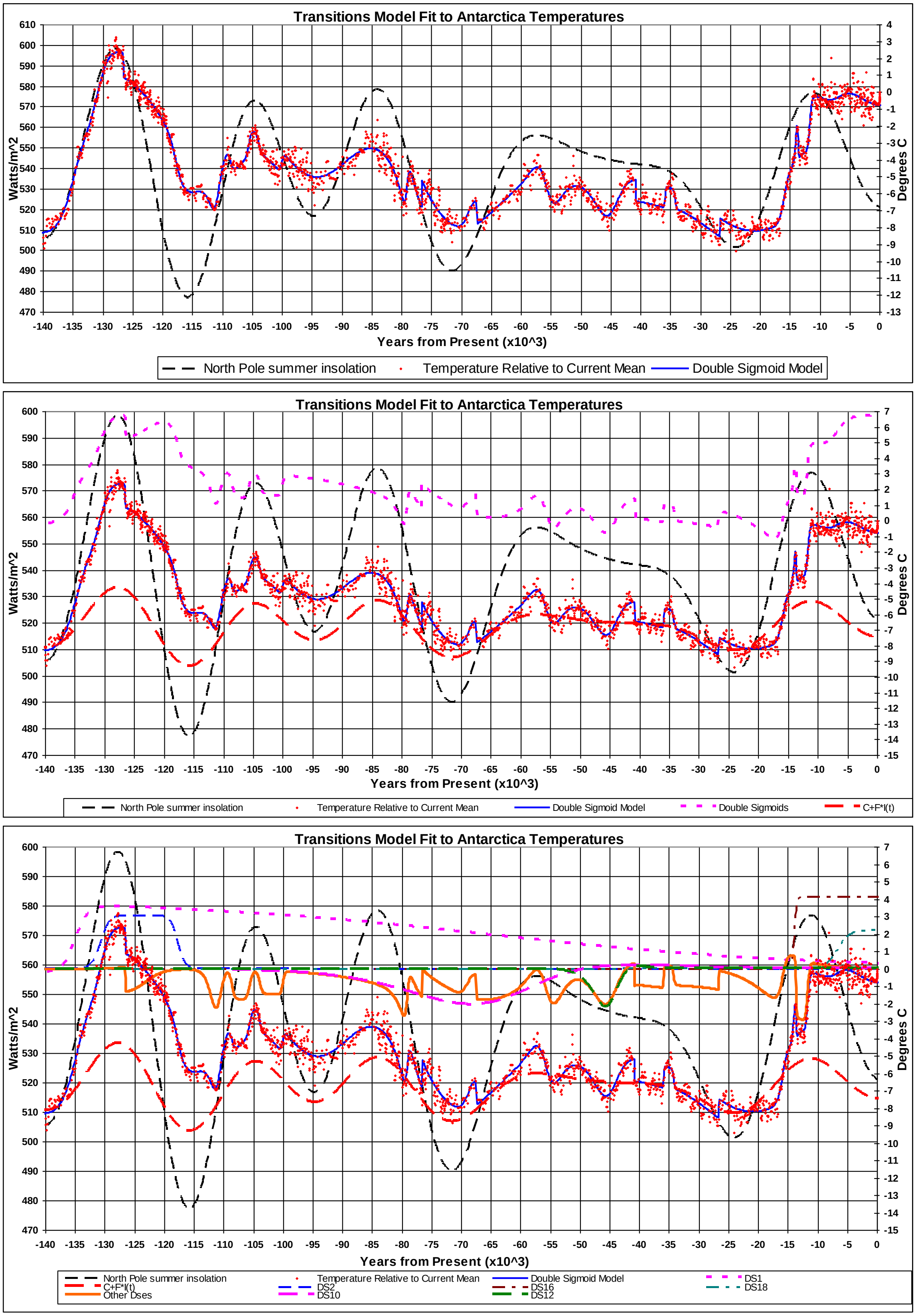


Figure II-10. The top graph shows the measured Antarctica temperatures for the Last Major Ice Age, including the rise into it. The fit to the temperature data is shown and the North Pole solar insolation is shown as a dotted line. The middle graph also shows the contributions to the fit by the North Pole insolation term with factor F (plus a constant C) and the contributions of the 18 double sigmoids. The bottom graph breaks out the double sigmoids into the four major ones and the other 14 ones.

The separate double sigmoids are shown in Figure II-11.

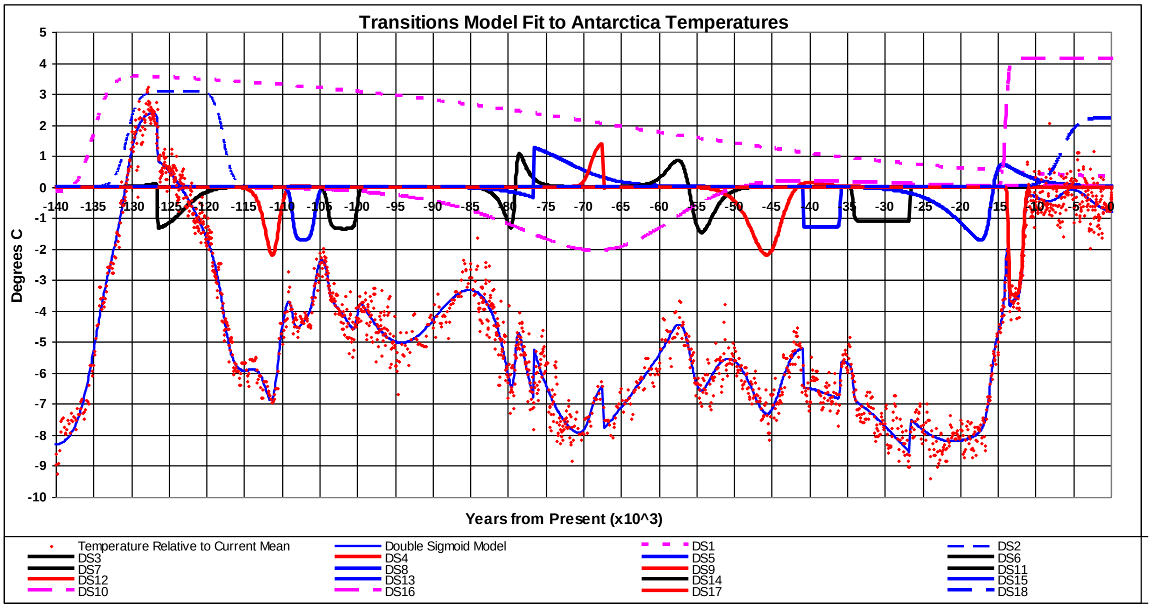


Figure II-11. The fit showing all of the 18 double-sigmoids used in the fit.

Figure II-12 shows the residual temperature for the Last Major Ice Age, including the rise into it, after the fit is subtracted from the measured temperatures.

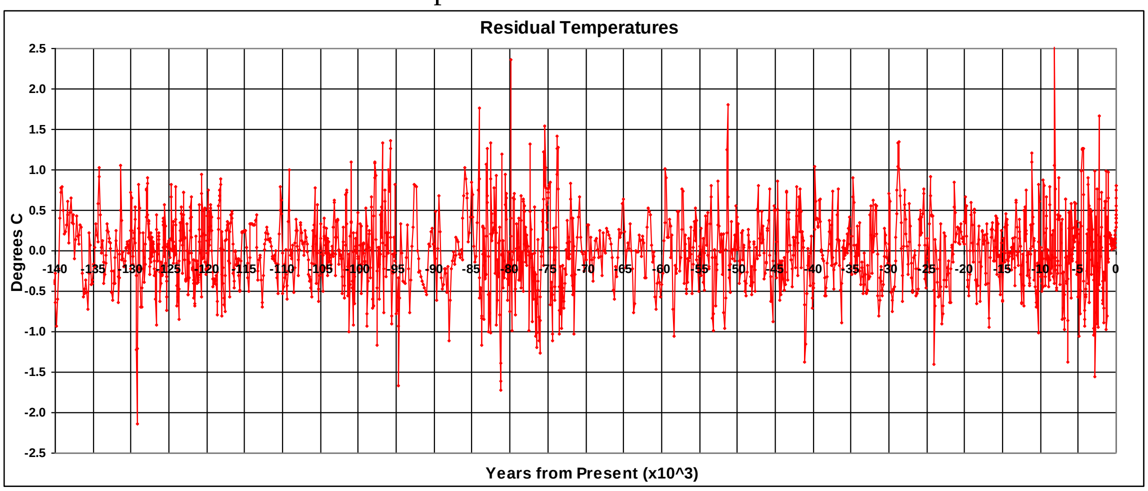


Figure II-12. Residual temperature for the Last Major Ice Age, including the rise into it, after the fit is subtracted from the measured temperatures.

Part III. Transitions Model Fits to Antarctica CO_2 Data

The Vostok Antarctica carbon dioxide data[1] can be downloaded from the Internet[12]. In this section fits are made to those data using the double-sigmoid transitions model.

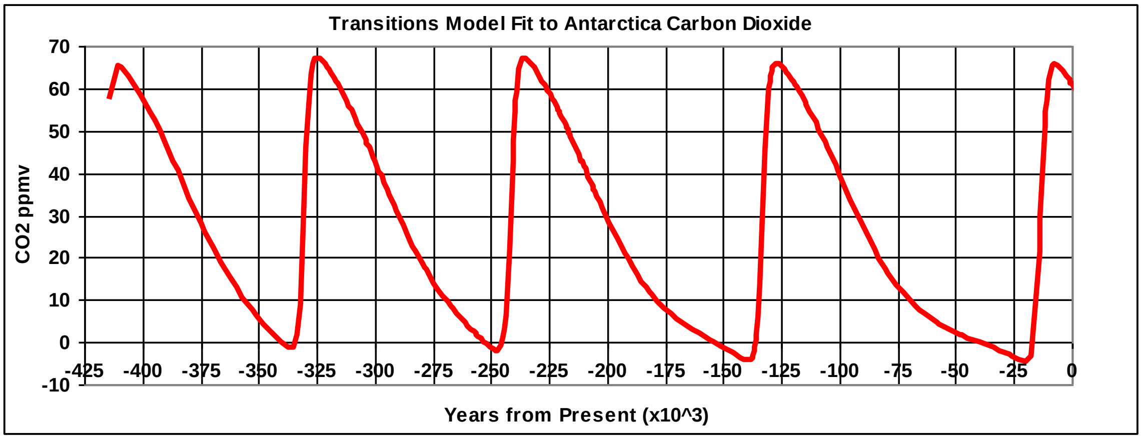
$\chi^2 = \sum_{i=1}^{N} (T_{mi} - T_{ci})^2 / 100$ is used for these fits, which corresponds to an error of 10 ppmv for the data.

Table III-1 gives the fit parameters and Figure III-1 shows the fit for one double sigmoid with the same parameters for all four past Major Ice Ages. Note the similarity of the graph of the double sigmoids in Figure III-1 to the double sigmoids for the temperature in Figure 3.

Note that the correlation with the summer North-Pole insolation (F parameter) is negative, instead of positive as it is for Antarctica temperatures. The fit is almost as good with F set equal to zero. Probably the CO_2 data need to be tuned to the insolation, similar to what was done with the temperature above, in order to use the insolation term in the fit.

Table III-1. Fit parameters for one double sigmoid with the same parameters for all four past Major Ice Ages for the double-sigmoid transitions model. The fit is to the Vostok Antarctica CO_2 data excluding the last 1000 years. (**Red** indicates variable parameters.) The data fitted were not tuned to the North-Pole insolation.

s_1 (ppmv)	c_{11} (kiloyears)	w_{11} (kiloyears)	c_{12} (kiloyears)	w_{12} (kiloyears)	F (ppmv/(Watts/m^1))	CO_2 (ppmv)
78.11	**-0.8212**	**2.629**	**32.37**	**31.45**	**-0.02895**	**215.3**
t_{3PMI} (kiloyears)	t_{2PMI} (kiloyears)	t_{PMI} (kiloyears)	t_{LMI} (kiloyears)	t_{CMI} (kiloyears)	t_{NMI} (kiloyears)	t_{2NMI} (kiloyears)
-415.6	**-329.4**	**-240.5**	**-131.9**	**-12.47**	112	230
Chi Square:	569	Data:	284	Parameters:	12	



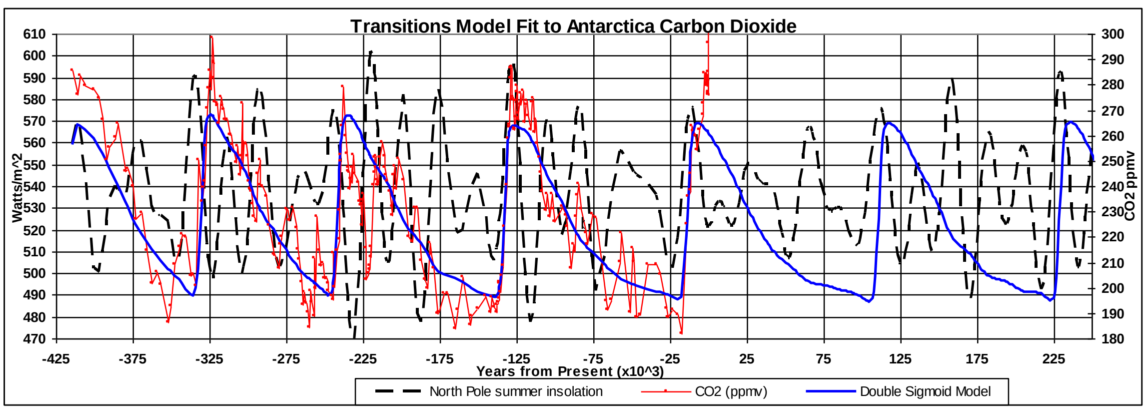


Figure III-1. Fit to the Vostok Antarctica carbon dioxide data, excluding the last 1000 years, for the double-sigmoid transitions model with the same parameters for all Major Ice Ages. The top graph shows the double-sigmoid.

Table III-2 gives the fit parameters and Figure III-2 shows the fit for two double sigmoids with the same parameters for all four past Major Ice Ages.

Table III-2. Fit parameters for two double sigmoids with the same parameters for all four past Major Ice Ages for the double-sigmoid transitions model. The fit is to the Vostok Antarctica CO_2 data excluding the last 1000 years. (**Red** indicates variable parameters.) The data fitted were not tuned to the North-Pole insolation.

s_1 (ppmv)	c_{11} (kiloyears)	w_{11} (kiloyears)	c_{12} (kiloyears)	w_{12} (kiloyears)	F (ppmv/(Watts/m^1))	CO_2 (ppmv)
47.04	**8.717**	**0.6462**	**56.81**	**15.52**	**-0.04632**	**223.2**
s_2 (ppmv)	c_{21} (kiloyears)	w_{11} (kiloyears)	c_{22} (kiloyears)	w_{22} (kiloyears)		
40.09	**4.362**	**1.702**	**15.78**	**9.082**		
t_{3PMI} (kiloyears)	t_{2PMI} (kiloyears)	t_{PMI} (kiloyears)	t_{LMI} (kiloyears)	t_{CMI} (kiloyears)	t_{NMI} (kiloyears)	t_{2NMI} (kiloyears)
-423.5	**-335.9**	**-248.2**	**-138.7**	**-19.88**	112	230
Chi Square:	503	Data:	284	Parameters:	17	

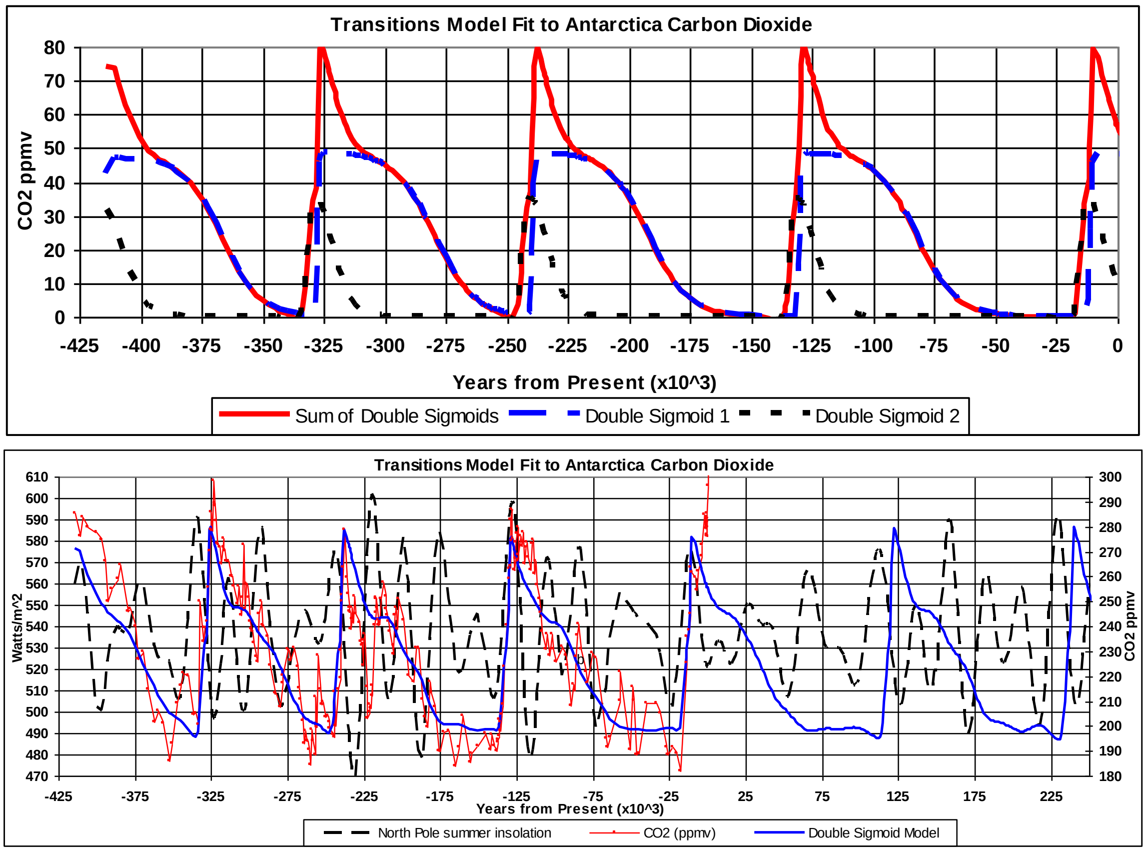
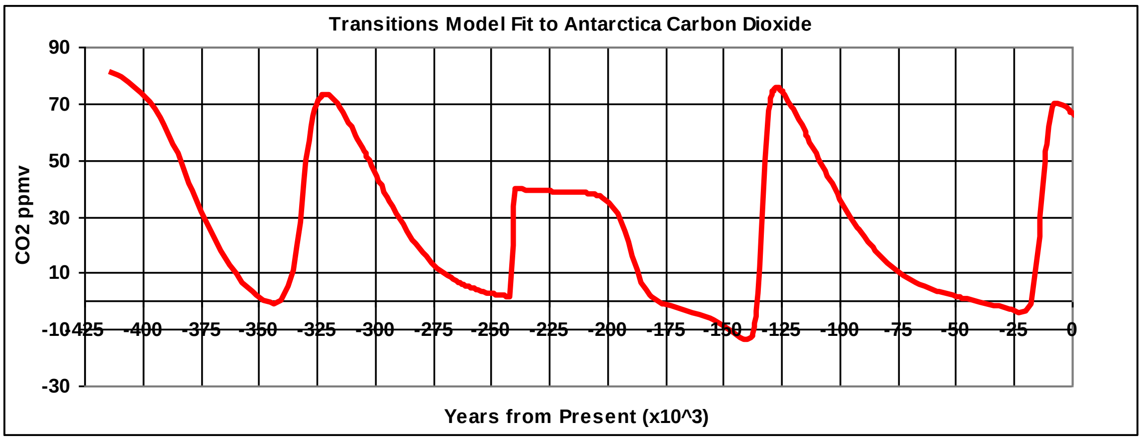


Figure III-2. Fit to the Vostok Antarctica carbon dioxide data, excluding the last 1000 years, for the two-double-sigmoid transitions model with the same parameters for all Major Ice Ages. The top graph shows the double-sigmoid.

Table III-3 gives the fit parameters and Figure III-3 shows the fit for one double sigmoids with different parameters for the four past Major Ice Ages. Note the similarity of the graph of the double sigmoids in Figure III-3 to the double sigmoids for the temperature in Figure 6.

Table III-3. Fit parameters for one double sigmoid with different parameters for each of the four past Major Ice Ages. The fit is to the Vostok Antarctica CO_2 data excluding the last 1000 years. (**Red** indicates variable parameters.) (The j index in the equation is suppressed; it is represented by the different rows) The data fitted were not tuned to the North-Pole insolation.

Double Sigmoid	s_1 (ppmv)	c_{11} (kiloyears)	w_{11} (kiloyears)	c_{12} (kiloyears)	w_{12} (kiloyears)	F (ppmv/(Watts/m^1))
DS1	**84.13**	**-1.176**	**0.1000**	**35.52**	**21.412**	**-0.08827**
DS2	**96.14**	**3.035**	**5.945**	**32.41**	**29.04**	C (ppmv)
DS3	**38.43**	**3.444**	**0.4236**	**53.74**	**7.206**	**248.7**
DS4	**105.7**	**-4.960**	**3.018**	**17.48**	**32.46**	
DS5	**83.48**	**-2.067**	**3.339**	32.68	31.57	
t_{3PMI} (kiloyears)	t_{2PMI} (kiloyears)	t_{PMI} (kiloyears)	t_{LMI} (kiloyears)	t_{CMI} (kiloyears)	t_{NMI} (kiloyears)	t_{2NMI} (kiloyears)
-415	-334	-244	-128	-11	112	230
Chi Square:	315	Data:	284	Parameters:	25	



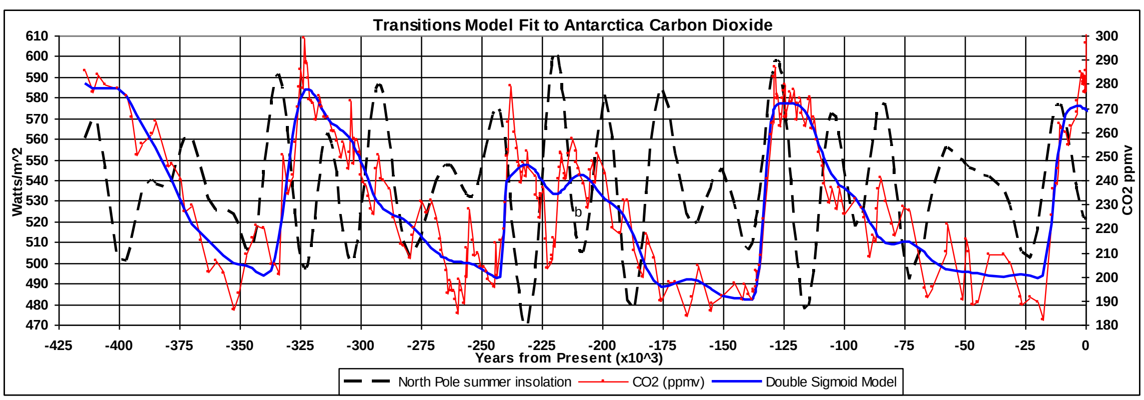


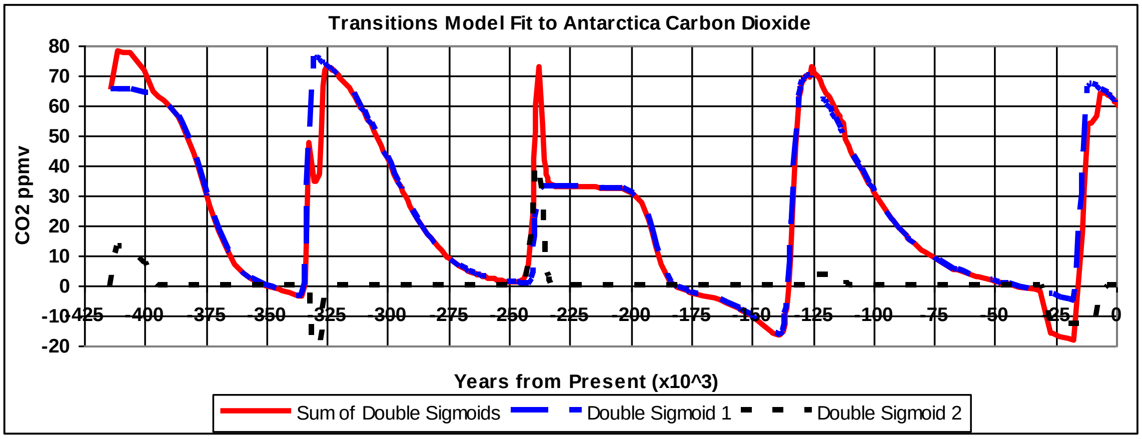
Figure III-3. Fit to the Vostok Antarctica carbon dioxide data, excluding the last 1000 years, for the one-double-sigmoid transitions model with different parameters for the Major Ice Ages. The top graph shows the double-sigmoid.

Table III-4 gives the fit parameters and Figure III-4 shows the fit for two double sigmoids with different parameters for the four past Major Ice Ages.

Table III-4. Fit parameters for two double sigmoids with different parameters for each of the four past Major Ice Ages. The fit is to the Vostok Antarctica CO_2 data excluding the last 1000 years. (**Red** indicates variable parameters.) (The j index in the equation is suppressed; it is represented by the different rows) The data fitted were not tuned to the North-Pole insolation.

Double Sigmoid	s_1 (ppmv)	c_{11} (kiloyears)	w_{11} (kiloyears)	c_{12} (kiloyears)	w_{12} (kiloyears)	F (ppmv/(Watts/m^1))

DS11	**65.47**	**-1.176**	**0.1**	**39.78**	**12.27**	**-0.07676**
DS12	**82.29**	**1.482**	**0.8034**	**34.49**	**23.86**	C (ppmv)
DS13	**32.86**	**4.764**	**0.9060**	**53.73**	**5.742**	**245.7**
DS14	**103.6**	**-4.999**	**2.905**	**14.41**	**33.10**	
DS15	**73.99**	**-3.361**	**1.920**	32.68	31.57	
Double Sigmoid	s_2 (ppmv)	c_{21} (kiloyears)	w_{21} (kiloyears)	c_{22} (kiloyears)	w_{22} (kiloyears)	
DS21	**12.73**	**3.188**	**0.09790**	**15.05**	**2.738**	
DS22	**-41.81**	**3.171**	**0.09622**	**7.067**	**0.9592**	
DS23	**91.24**	**4.959**	**2.434**	**7.051**	**1.892**	
DS24	**3.721**	**2.176**	**0.0420**	**16.42**	**0.1431**	
DS25	**-12.91**	**-16.42**	**0.1**	**3.606**	**1.158**	
t_{3PMI} (kiloyears)	t_{2PMI} (kiloyears)	t_{PMI} (kiloyears)	t_{LMI} (kiloyears)	t_{CMI} (kiloyears)	t_{NMI} (kiloyears)	t_{2NMI} (kiloyears)
-415	-334	-244	-128	-11	112	230
Chi Square:	260	Data:	284	Parameters:	50	



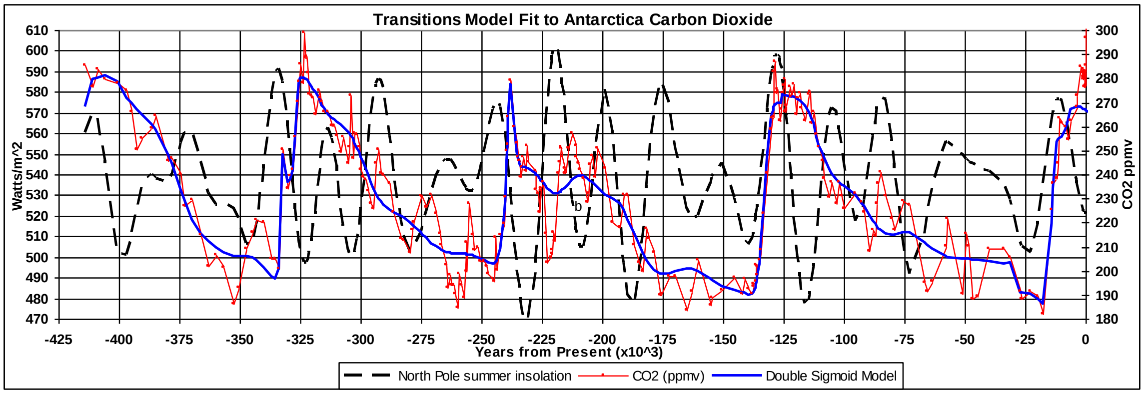


Figure III-4. Fit to the Vostok Antarctica carbon dioxide data, excluding the last 1000 years, for the two-double-sigmoid transitions model with different parameters for the Major Ice Ages. The top graph shows the double-sigmoid.

Appendix I. Double Sigmoids

A single sigmoid is

$$y = \frac{1}{1+\exp\left(-2\dfrac{x-c}{w}\right)} = \frac{1}{2}\left[1+\tanh\left(\frac{x-c}{w}\right)\right] \text{ with constraint } w > 0.$$

Figure AI-1. shows a sigmoid along with a double sigmoid.

The TableCurve[13] program gives the following equation for the asymmetric double sigmoid:

$$y = \frac{a}{1+\exp\left(-\dfrac{x-b+c/2}{d}\right)}\left[1-\frac{1}{1+\exp\left(-\dfrac{x-b-c/2}{e}\right)}\right]$$

with constraints $c > 0$, $d > 0$ and $e > 0$.

This is a cumbersome equation and computational difficulties can occur when using it because of the infinite asymptotic property of the exponential with positive argument. (The symmetric double sigmoid given by TableCurve is even more complicated looking.)

A simpler and computational better formulation is:

$$y = \frac{1}{2}\left[\tanh\left(\frac{x-c_1}{w_1}\right) - \tanh\left(\frac{x-c_2}{w_2}\right)\right]$$

with constraints $w_1 > 0$ and $w_2 > 0$.

This is the equation used in this paper. (The symmetric version is simply the case where $w_1 = w_2 = w$.)

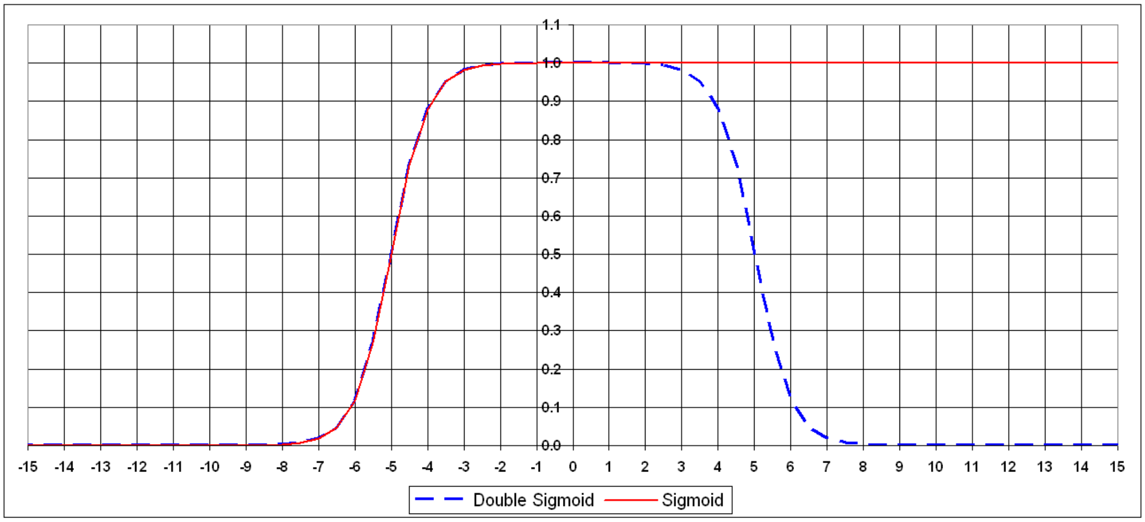


Figure AI-1. A single sigmoid and a symmetrical double sigmoid for parameters $c = c_1 = -5, c_2 = 5$ and $w = 1$.

Figure AI-2. shows some double sigmoids for different values of the second w parameter. Note that, when the second width (w_2) becomes comparable to the distance between c_1 and c_2, the second sigmoid causes the first sigmoid to dip down below the zero axis.

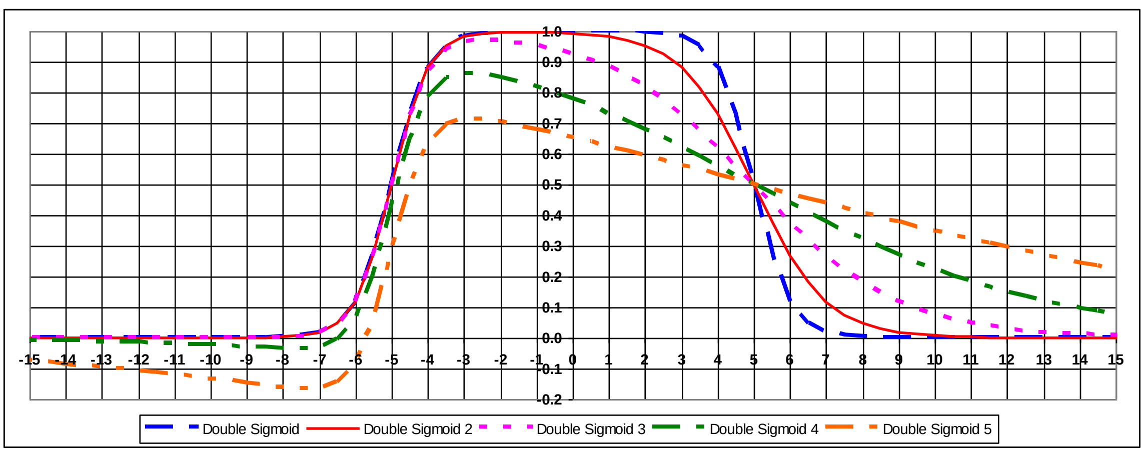
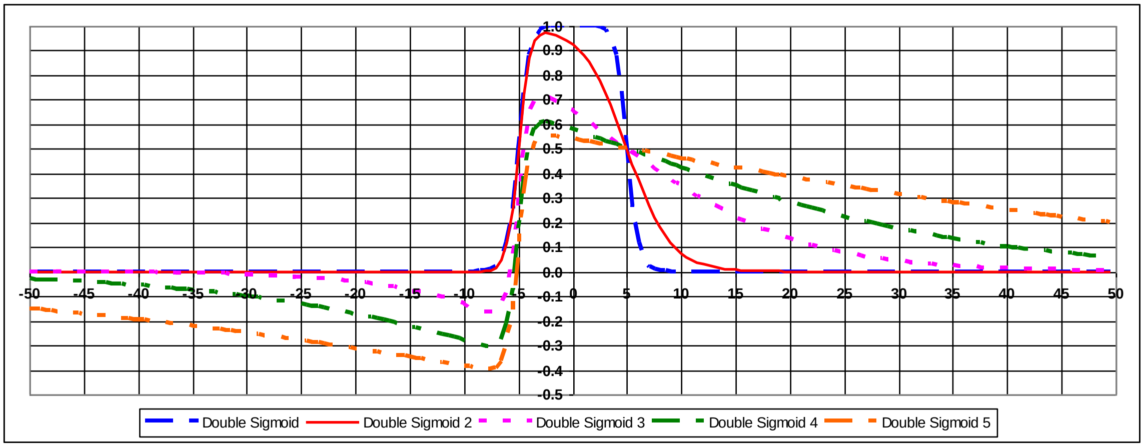


Figure AI-2. Double sigmoids for different amounts of asymmetry. The parameters are $c = c_1 = -5, c_2 = 5, w_1 = 1$ and $\mathrm{w}_2 = (1, 2, 4, 8, 16)$.

This effect is shown more clearly in Figure AI-3. for larger asymmetry. In the case shown, because the second part of the double sigmoid has such a large width, w, it actually starts occurring first, the first part of the double sigmoid, with its much smaller width starts reversing the second part of the double sigmoid before the second part has reached its asymptote. Eventually the first part of the double sigmoid asserts itself and causes a switch from a turning off to a turning on of the state. As shown in the bottom half of Figure AI-3. for a negative strength factor, the event could be reversed in sign, mostly representing a turning off of a state.



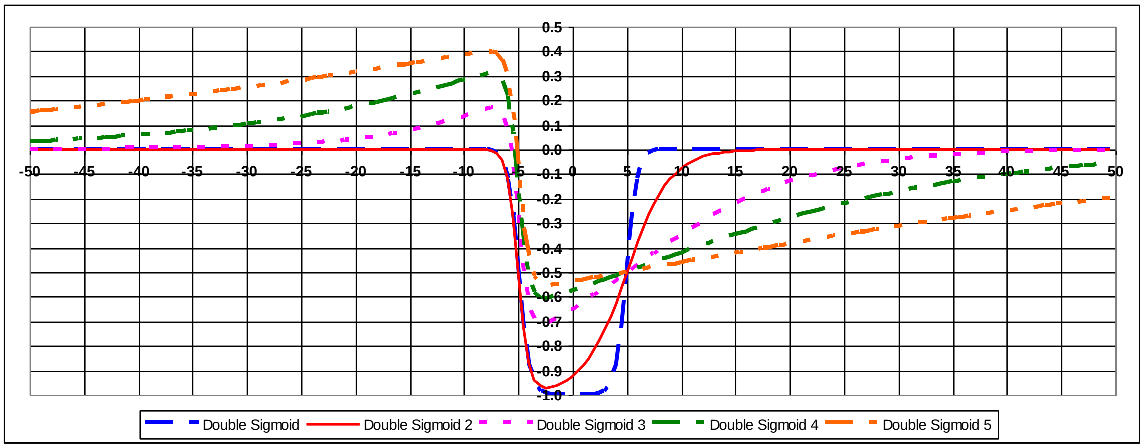


Figure AI-3. Double sigmoids for different amounts of asymmetry. The parameters are $c = c_1 = -5$, $c_2 = 5$, $w_1 = 1$ and $\mathrm{w}_2 = (1, 4, 16, 32, 64)$. The bottom graph is for turning off and back on an Earth state.

Appendix II. Solar Insolation Alone

It might be of interest to see well solar insolation alone can fit the tuned Antarctica temperatures. The equation to use in the fit is $T(t) = C + F \cdot I(t)$. The best fit for all of the last four Major Ice Ages is for $C = -34.16\ ^{\circ}C$ and $F = 0.05475 \dfrac{^{\circ}C}{Watts/m^2}$. The best fit for the Last Major Ice Age plus the rise into it, but excluding the last 1000 years is for $C = -37.78\ ^{\circ}C$ and $F = 0.06268 \dfrac{^{\circ}C}{Watts/m^2}$. These fits are not very good, as can be seen in Figure AII-1.

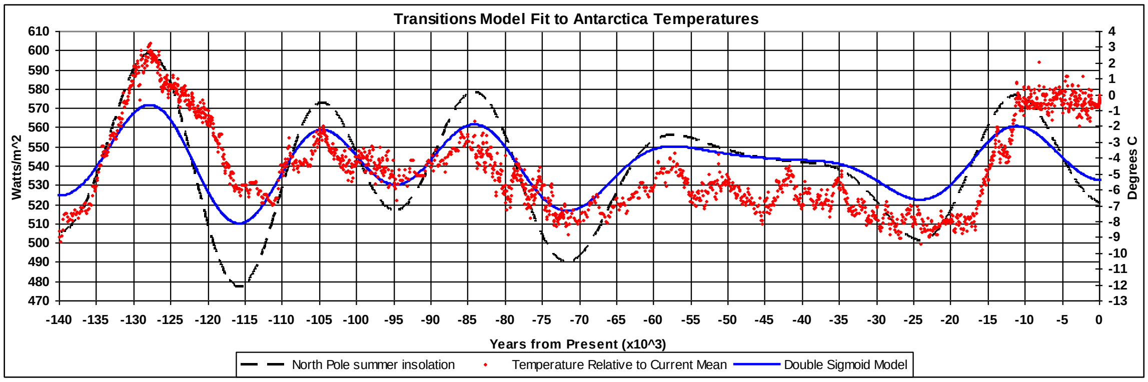


Figure AII-1. Fit of the solar insolation alone to the tuned Antarctica temperature data for the Last Major Ice Age. The dashed curve is the solar insolation and the solid curve is the fit to the data.

Acknowledgements

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13 March 2023

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[2] http://www.ngdc.noaa.gov/paleo/icecore/antarctica/vostok/vostok_isotope.html .
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[6] http://xml.gsfc.nasa.gov/archive/catalogs/6/6063/index_long.html and http://www.astr.ucl.ac.be/research/astro.html for Unix machines; http://www.roperld.com/science/InsolationCodes.htm for Microsoft Windows machines.
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[12] http://www.ngdc.noaa.gov/paleo/icecore/antarctica/vostok/vostok_isotope.html
[13] http://www.systat.com/products/TableCurve2D