Global Warming Mathematics

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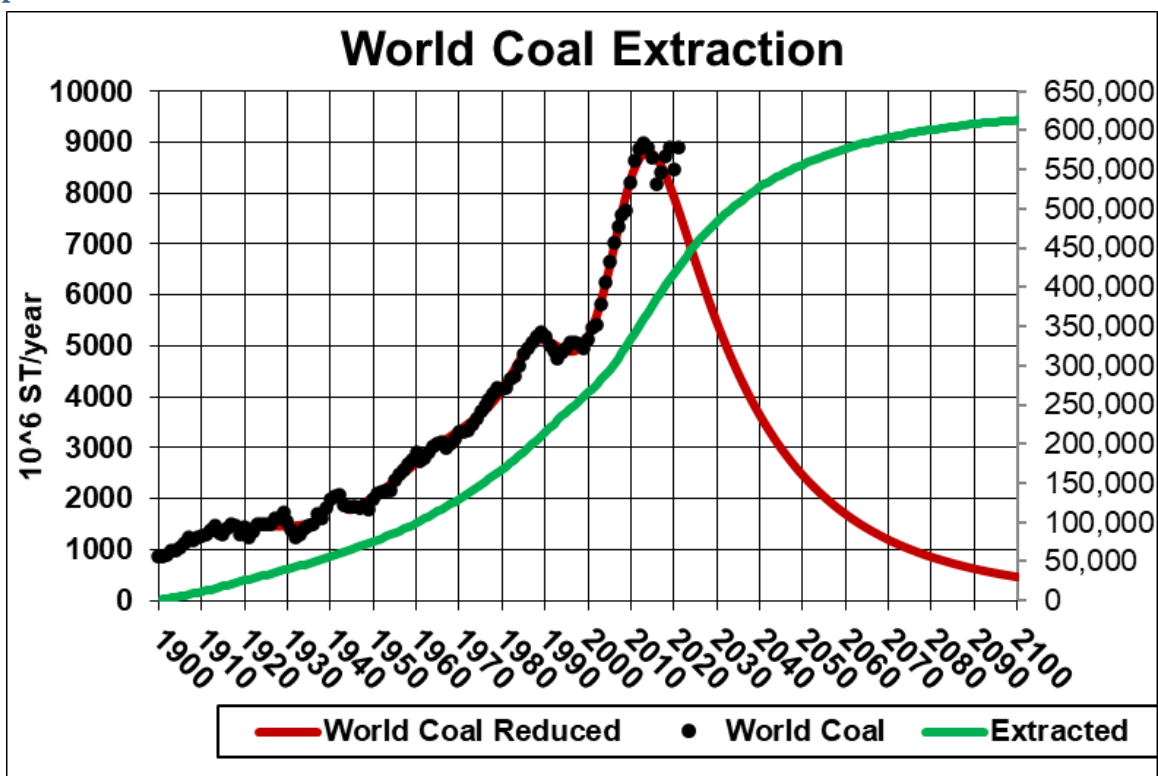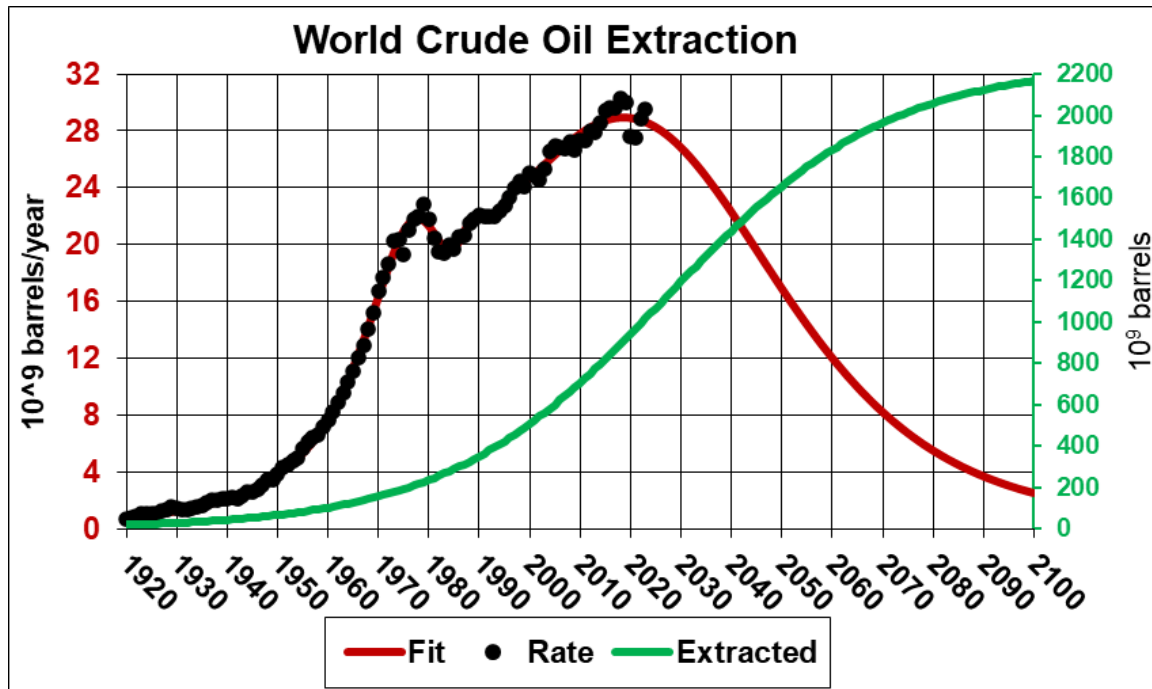http://arts.bev.net/RoperLDavid/

http://www.roperld.com/science/globalwarmingmathematics.pdf

Depletion Equation

$$P(t) = \frac{Q_\infty}{n\tau} \frac{(2^n - 1)\exp\left(\dfrac{t - t_{\frac{1}{2}}}{\tau}\right)}{\left[1 + (2^n - 1)\exp\left(\dfrac{t - t_{\frac{1}{2}}}{\tau}\right)\right]^{\frac{n+1}{n}}}$$
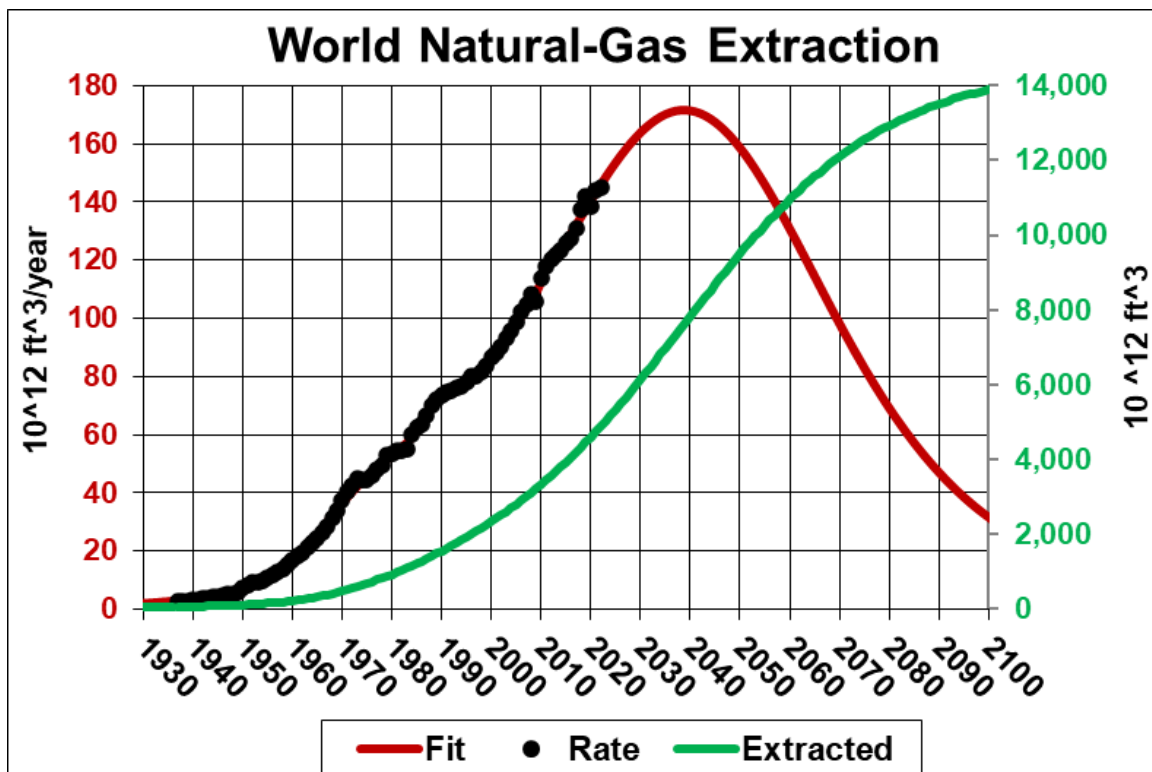
Coal Depletion

Crude-Oil Depletion

Natural-Gas Depletion

Carbon Emissions

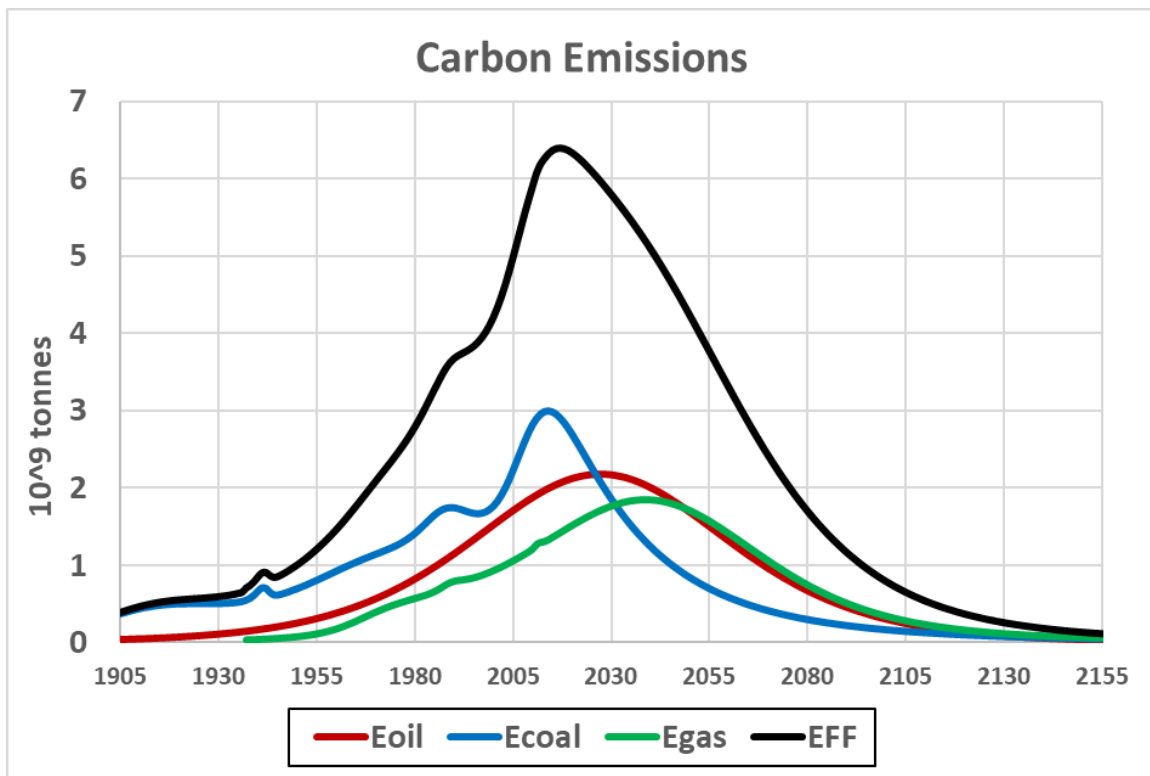The carbon emissions for fossil fuels are:

$$E_{Coal}(t) = 10^{-3} \cdot 0.907 \cdot 0.5 \cdot 0.75 P_{Coal}(t) \, ,$$

$$E_{Oil}(t) = 0.136 \cdot 0.84 \cdot 0.75 P_{Oil}(t) \text{ and}$$

$$E_{Gas}(t) = 0.0189 \cdot 0.76 \cdot 0.75 P_{Gas}(t),$$

assuming that 75% of each of the three fossil fuels is burned for fuel. Thus,

$$E_{FF}(t) = E_{Coal}(t) + E_{Oil}(t) + E_{Gas}(t).$$

The assumption is made here that the fossil fuels are burned in the year they are extracted.

Carbon-Dioxide Concentration in the Atmosphere

$$C_{FF}(t_n) = 0.47 \sum_{i=1}^{n} E_{FF}(t_i) \left[0.0875 \exp\left(\frac{t_n - t_i}{35600}\right) + 0.165 \exp\left(\frac{t_n - t_i}{682}\right) + 0.733 \exp\left(\frac{t_n - t_i}{50.8}\right) \right]$$

for each year t_n. The factor 0.47 is the canonical factor for converting carbon emissions to carbon-dioxide concentration in the atmosphere. The three exponentials represent the carbon-dioxide residence time. Below, the possibility of this factor increasing with concentration will be considered.
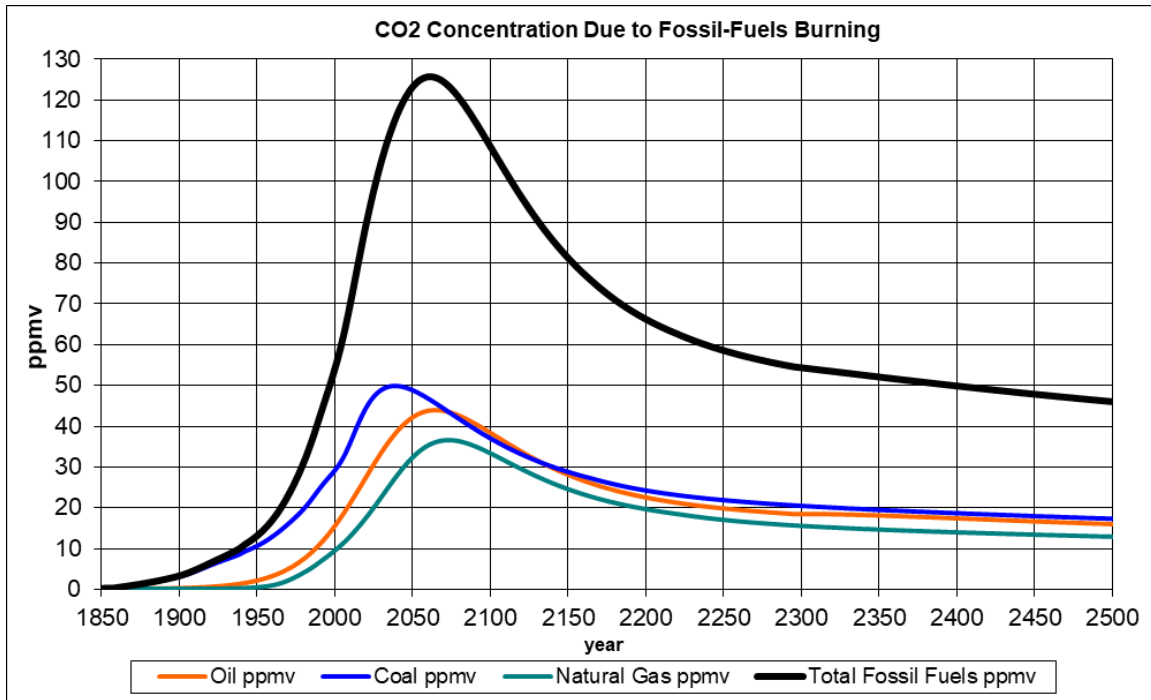
Note that the carbon-dioxide peaks are several decades later the extraction peaks, due to the carbon-dioxide residence time.

Before fossil fuels were starting to be burned for energy, there was already a background concentration of carbon dioxide in the atmosphere of about 280 ppmv. Adding that background to the carbon-dioxide concentration due to burning fossil-fuels yields:

**CO2 Concentration Due to Fossil-Fuels Burning + Non-Fossil-Fuels + Background**
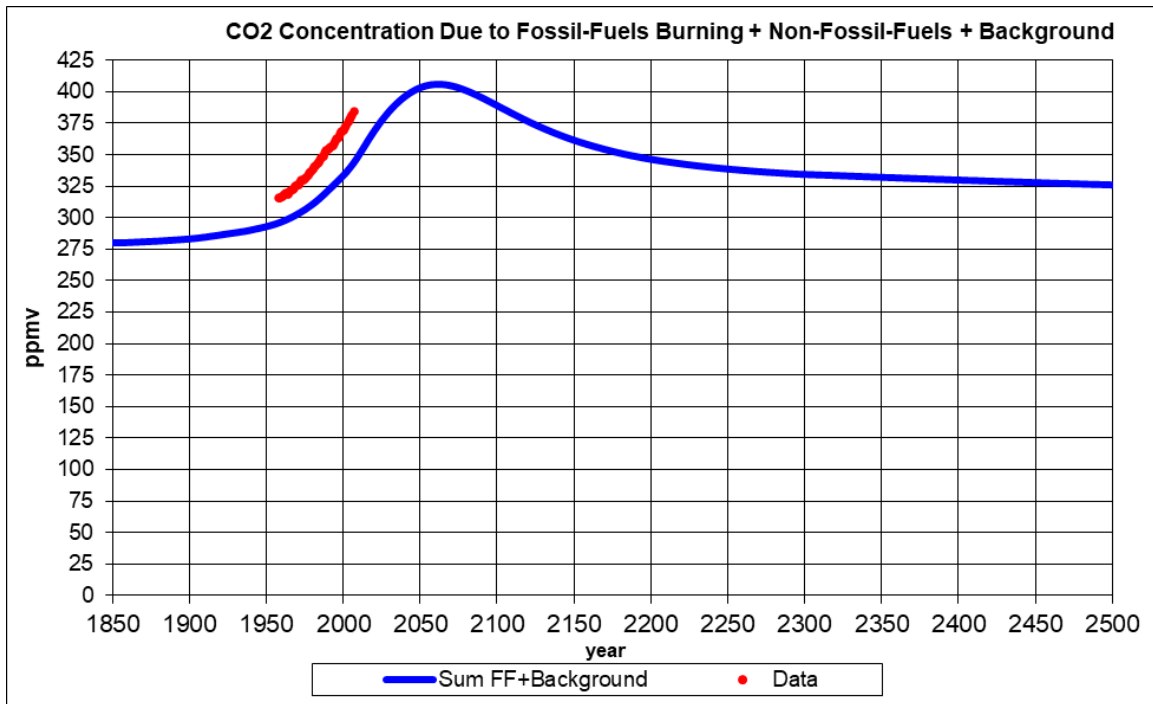
Note that the measured data are above the fossil-fuels concentration, as it should be because there are other sources of carbon emissions, such as deforestation.

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Non-Fossil-Fuels Emissions

Possibly a good approximation for non-fossil-fuels (NFF) emissions is that its contribution to carbon-dioxide concentration is proportional to the fossil-fuels concentration. Rational for this approach is that the amount of deforestation, soil depletion, etc. is probably proportional to the amount of fossil fuels burned. Without fossil fuels these activities could not occur with the intensity that they do occur.

A linear fit yields a background concentration of about 292 ppmv, well above 280 ppmv, the known background concentration. It is assumed that this is due to pre-fossil-fuels emissions due to intensive agriculture and early industrialization. A hyperbolic tangent is used to represent the pre-fossil-fuels rise from c1700 to c1900:

$$C(t_n) = 280 + \frac{300-280}{2}\left[1+\tanh\left(\frac{t_n-1851.8}{36.4}\right)\right] + 1.129 C_{FF}(t_n) + 0.5\left[C_{FF}(t_n)\right]^2/100$$
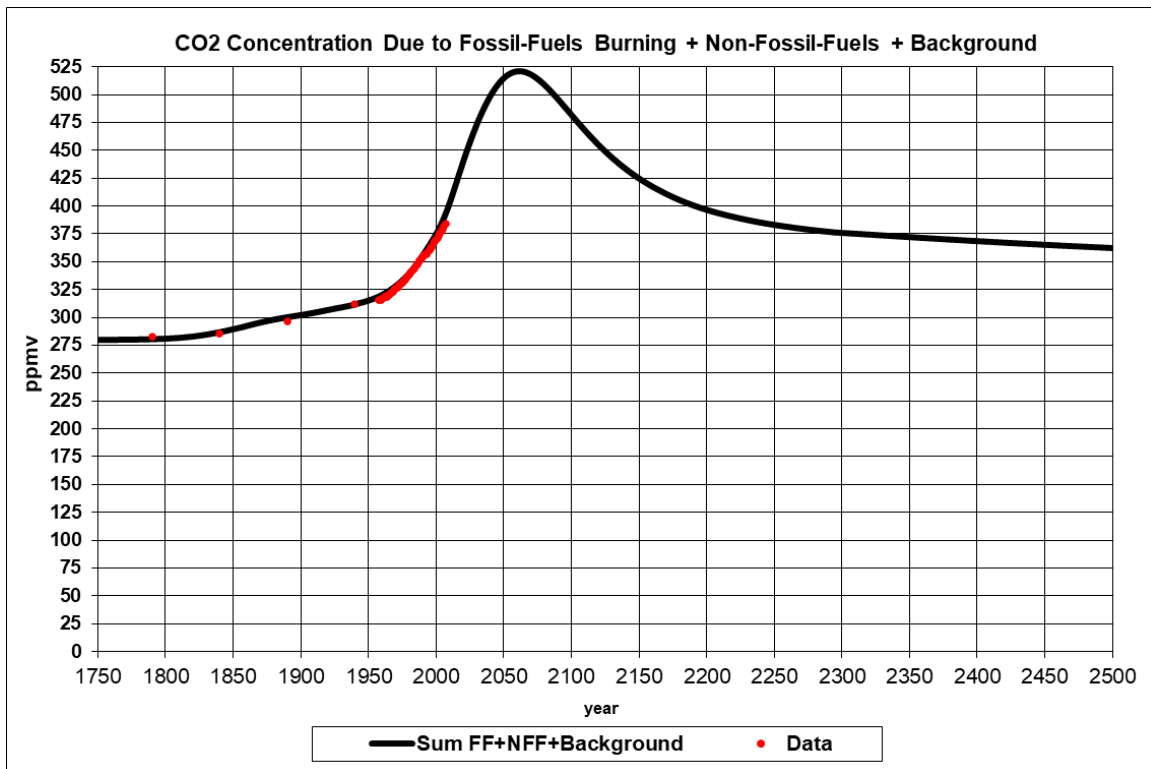
Temperature Change Due to Carbon Dioxide Concentration in the Atmosphere

I use a climate sensitivity of 3 degrees celsius for CO_2 concentration doubling to calculate the approximate temperature change relative to year 1700, dT (in Celsius degrees), due to a change in CO_2 concentration, C_i (in ppmv). The equation is:

$$dT(t_n) = \frac{3}{\ln 2} \ln\left(\frac{C(t_n)}{C_0}\right)$$

for year t_n relative to the concentration C_0 for a reference year (1700 AD). The s=3 factor is the climate sensitivity that makes the climate sensitivity factor be for doubling of the CO_2 concentration in the atmosphere, as it is defined. (Climate sensitivity is supposed to include the effects of fast feedback mechanisms, both positive and negative, in other variables caused by changing the CO_2 concentration.)

An equivalent version of the equation is

$$dT(t_n) = \frac{3}{\ln 2} \sum_{i=1}^{n} \ln\left(\frac{C(t_i)}{C(t_{i-1})}\right), \text{ where } C(t_0) = C_0.$$

Besides the climate sensitivity, one should allow for the fact that the ocean absorbs much of the solar energy impinging on the Earth and then slowly releases some of the heat energy to the atmosphere. There is a climate response function that represents the atmospheric temperature time delay after carbon is inserted into the atmosphere. The function is
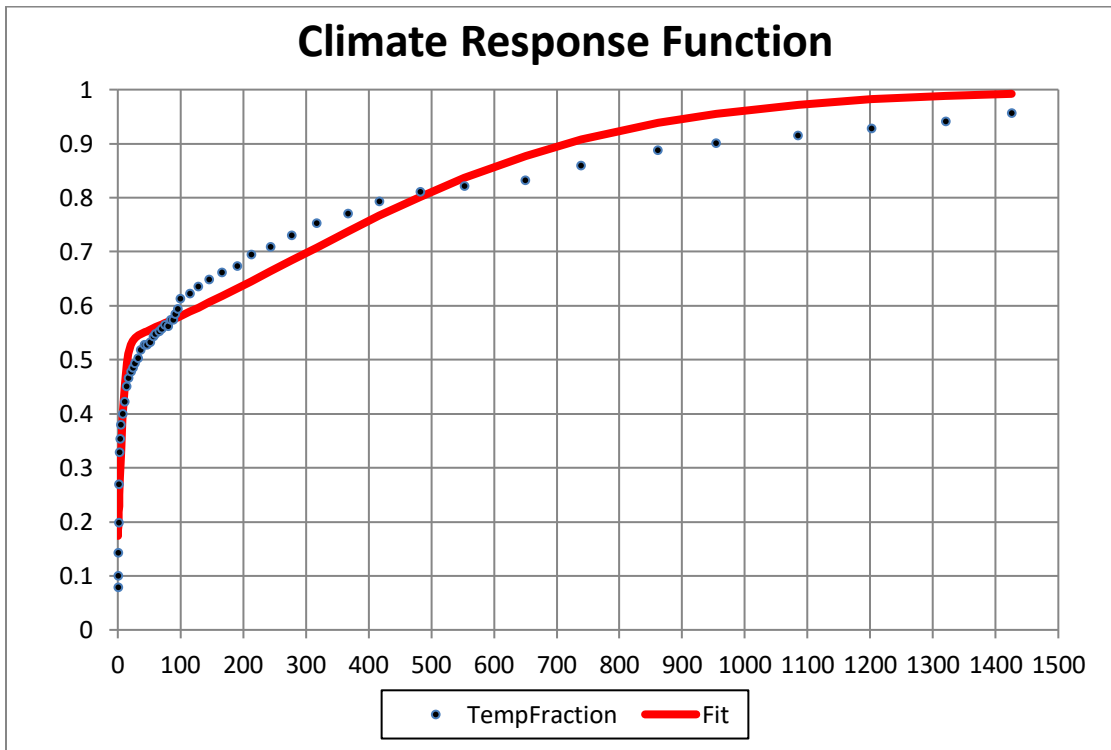

The two-hyperbolic-tangent fit is good because the data errors are large. The function is

$$0.368 \tanh\left(\frac{t}{10.5}\right) + \frac{0.632}{2}\left[1 + \tanh\left(\frac{t - 277}{524}\right)\right]$$

A better fit could be obtained with three hyperbolic tangents, but the errors on the data do not justify it.

The resulting temperature change relative to 1700 due to fossil-fuels burning and non-fossil-fuels emissions is:

$$dT(t_n) = \frac{3}{\ln 2} \sum_{i=1}^{n} \left\{ 0.368 \tanh\left(\frac{t_n - t_i}{10.5}\right) + \frac{0.632}{2}\left[1 + \tanh\left(\frac{t_n - t_i - 277}{524}\right)\right] \ln\left(\frac{C(t_i)}{C(t_{i-1})}\right) \right\}$$
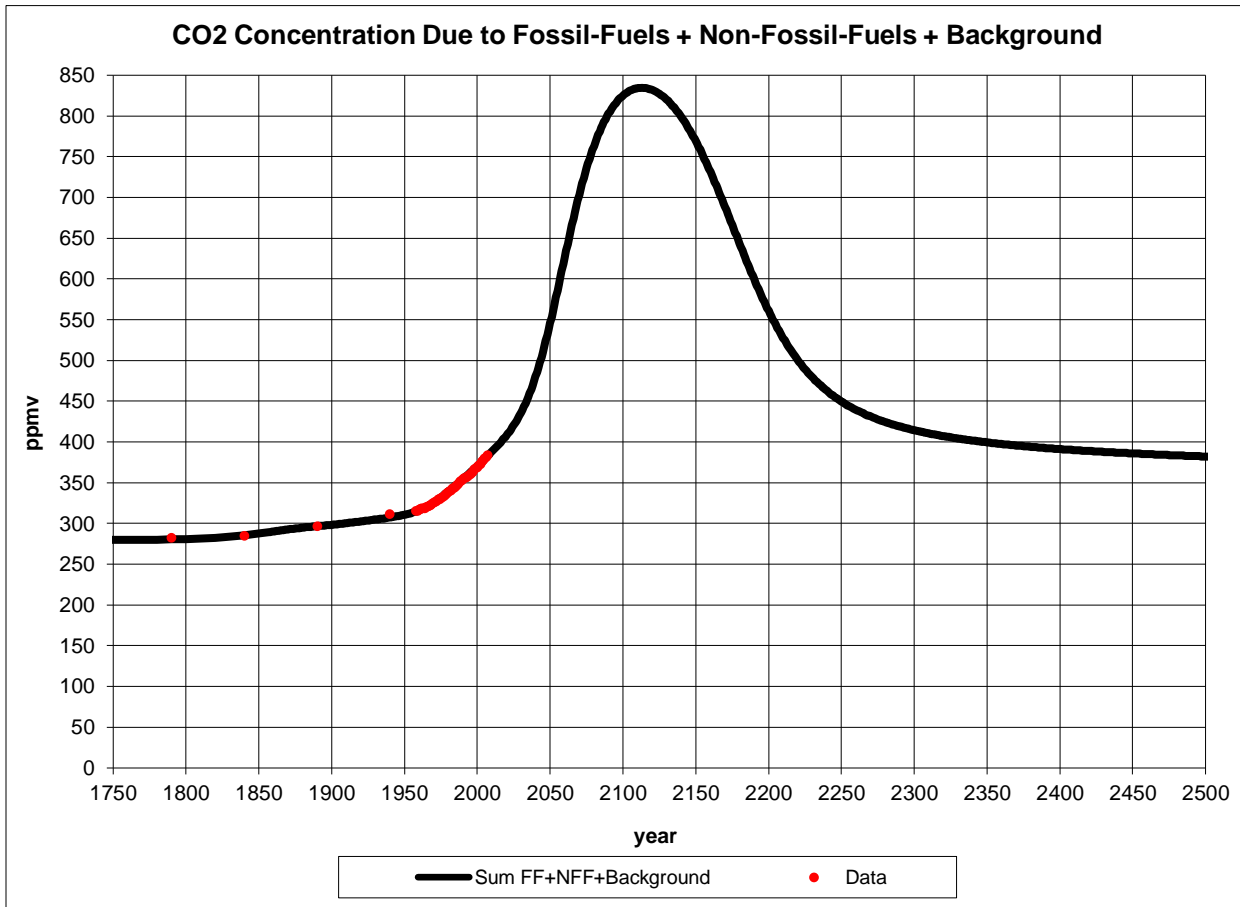
Temperature Change from 1700
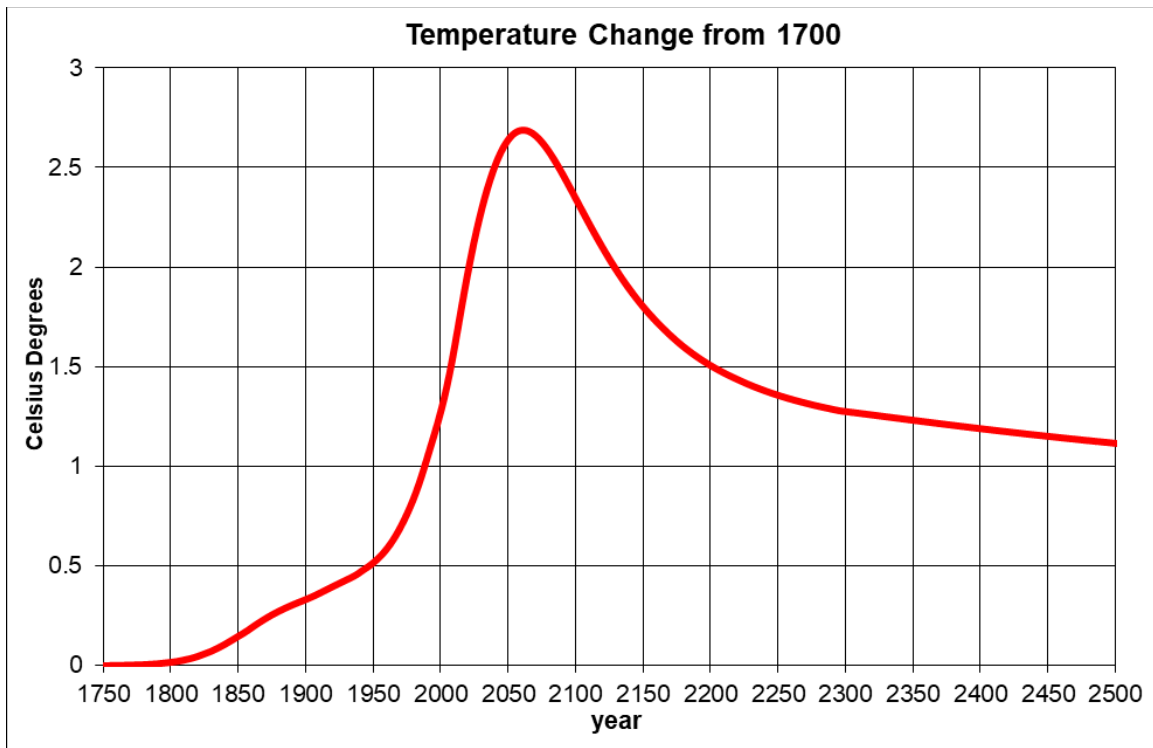
An unknown amount of global dimming should be subtracted from this curve; it is probably only a few tenths of a degree or smaller and quite variable with time.

The peak would be higher if it were not for the time delay between carbon emission and the atmospheric temperature. The asymptotic temperature is the same for both calculations, even though the ocean keeps some of the energy, because the climate sensitivity accounts for that.

# Temperature Change for High Carbon-Dioxide Concentration

Suppose that the oceans and the vegetation reach a saturation point for high carbon-dioxide concentration such that the 0.47 factor doubles to 0.94 according to a hyperbolic tangent centered at 450 ppmv with a width of 25 ppmv. Then the concentration will be

$$C_{Double}\left(t_n\right) = \frac{C\left(t_n\right)}{2}\left[3 + \tanh\left(\frac{C\left(t_n\right) - 450}{25}\right)\right]$$

CO2 Concentration Due to Fossil-Fuels + Non-Fossil-Fuels + Background

The temperature would be

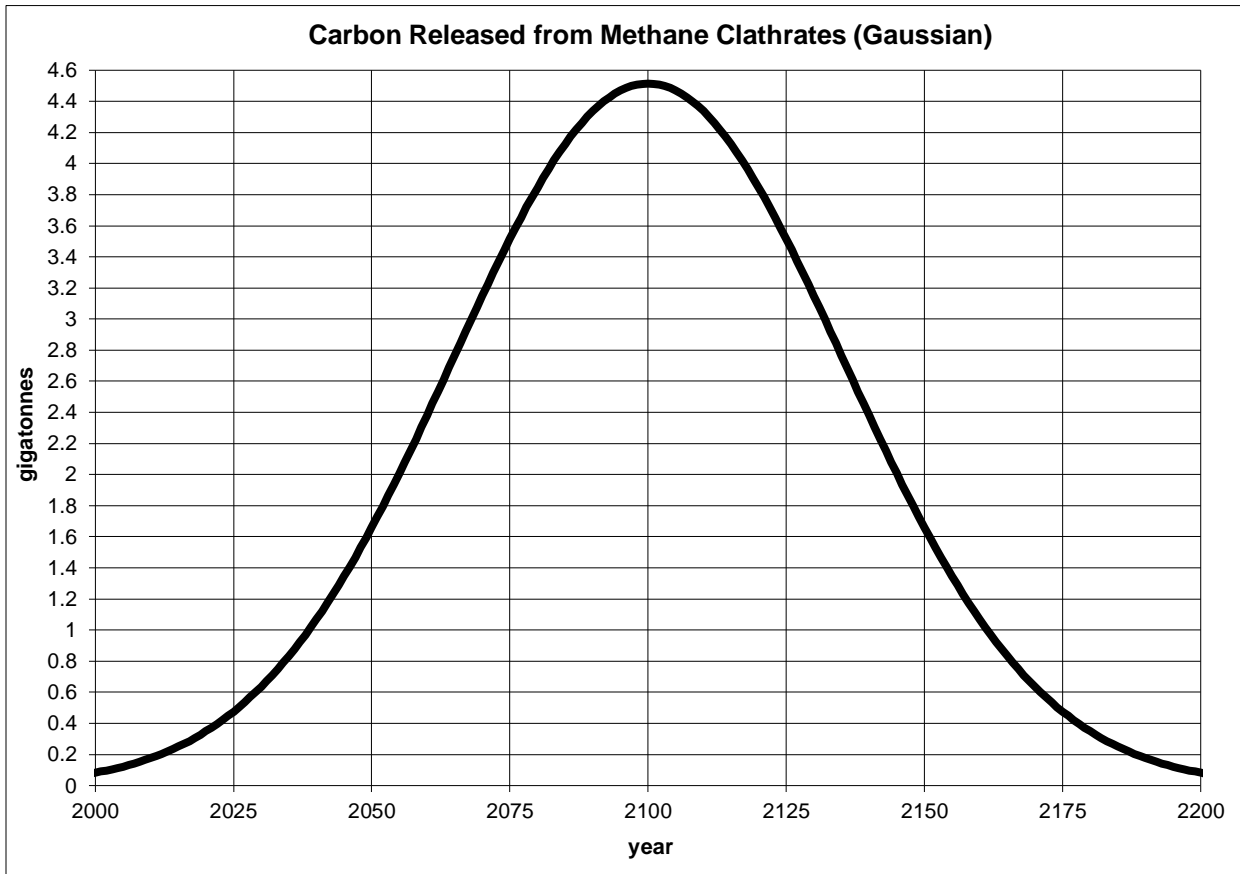$$dT_{Double}\left(t_n\right) = \frac{3}{\ln 2} \sum_{i=1}^{n} \left\{ 0.368\tanh\left(\frac{t_n - t_i}{10.5}\right) + \frac{0.632}{2}\left[1 + \tanh\left(\frac{t_n - t_i - 277}{524}\right)\right]\ln\left(\frac{C_{Double}\left(t_i\right)}{C_{Double}\left(t_{i-1}\right)}\right)\right\}$$

Temperature Change from 1700
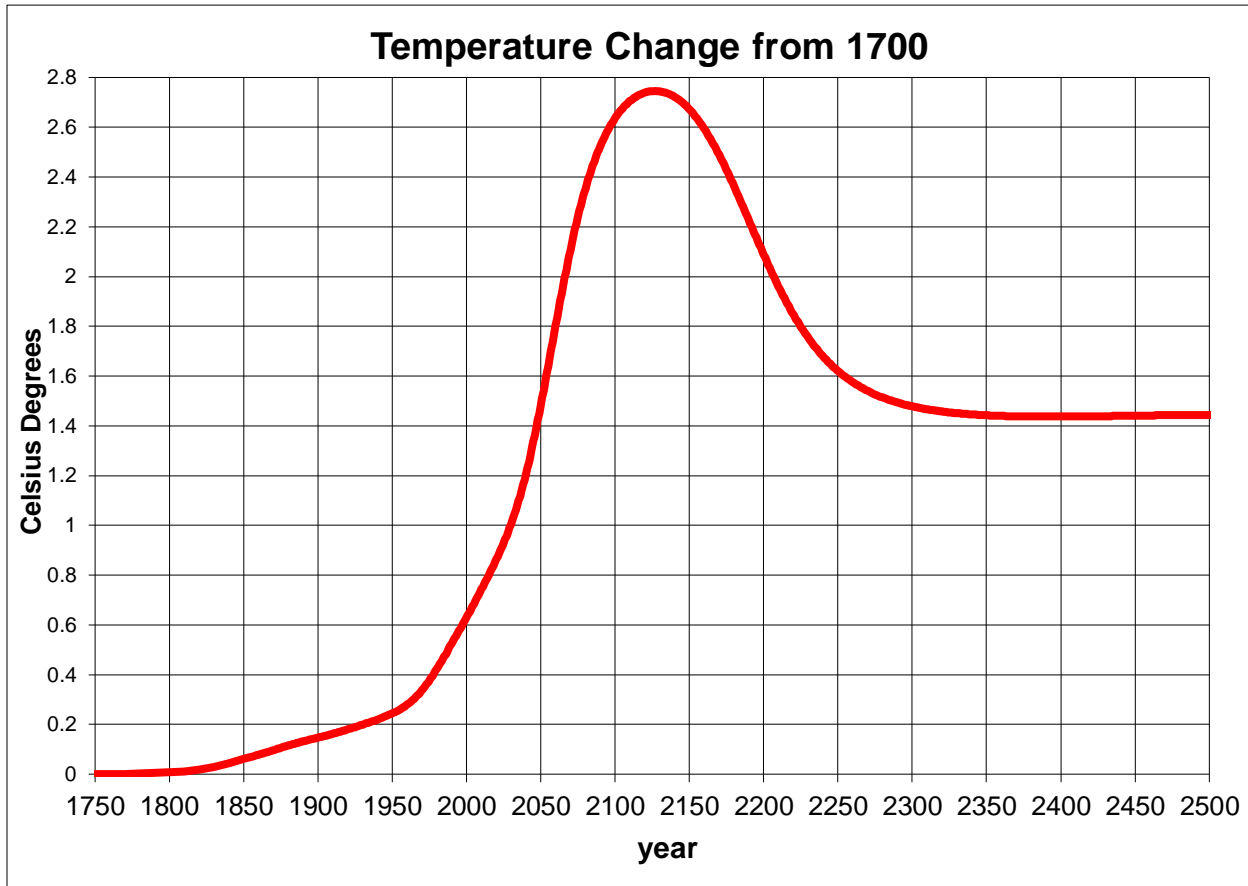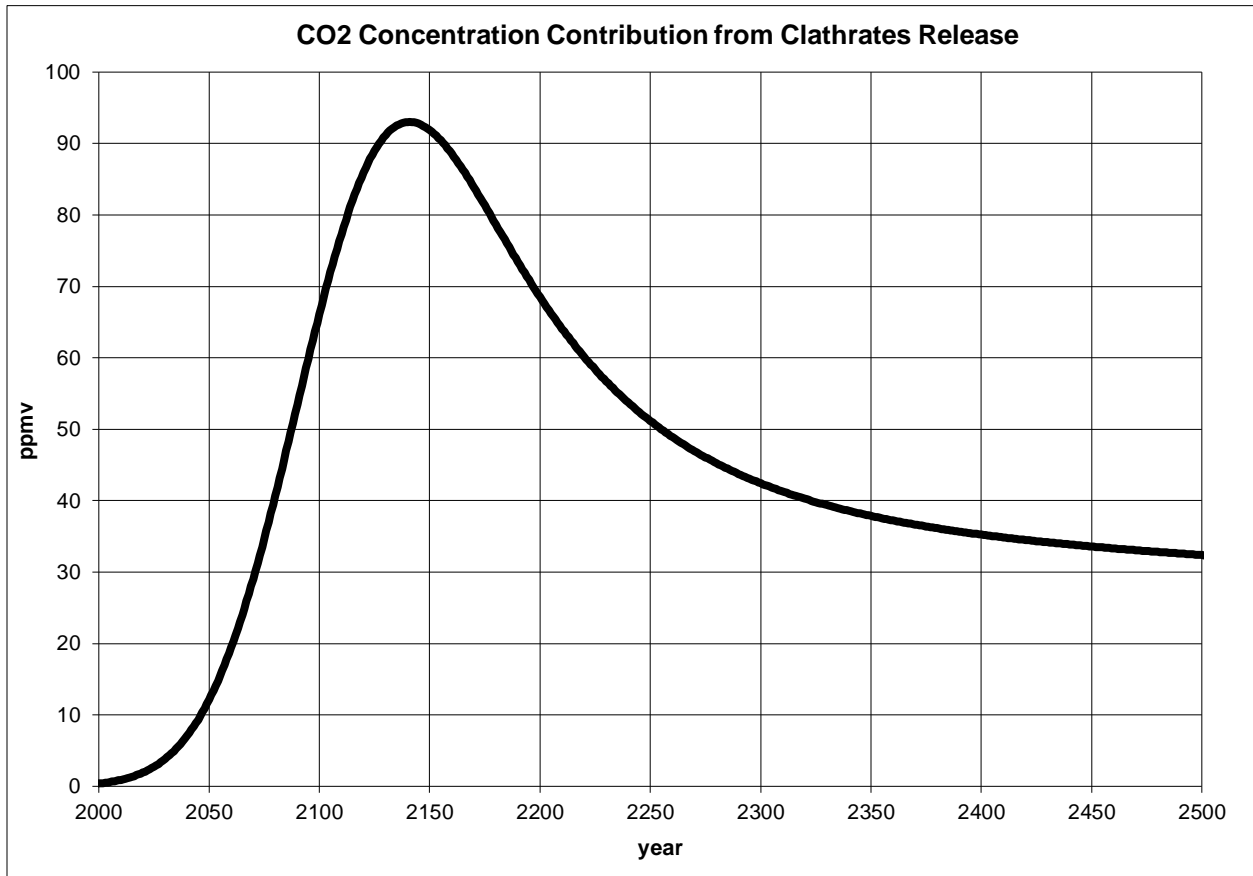
Carbon Release in the Arctic

Assume that global warming causes a large amount of carbon to be released into the atmosphere in the Arctic according to

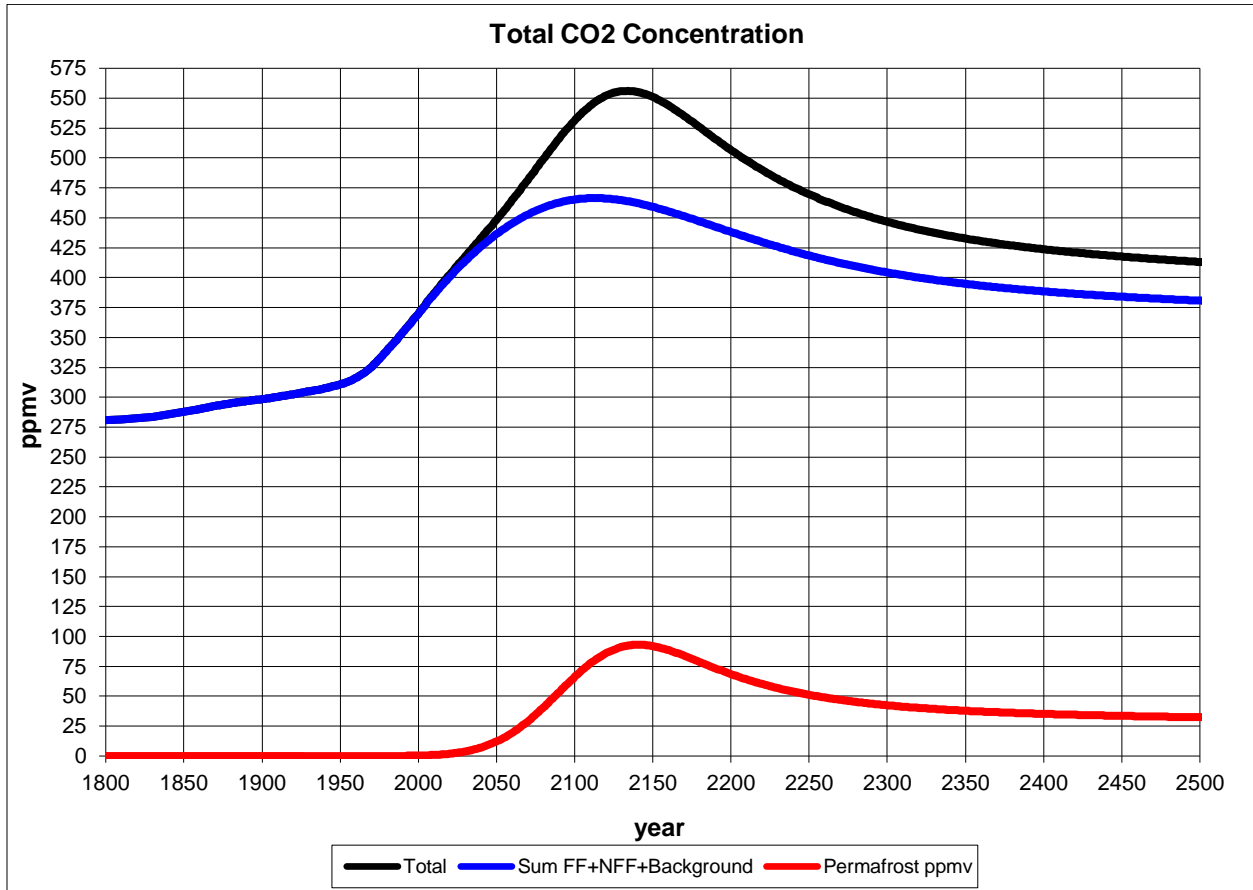$$E_{Arctic}(t) = 4.514 \exp\left[-\left(\frac{t-2100}{50}\right)^2\right]$$

Carbon Released from Methane Clathrates (Gaussian)

The contribution to the carbon-dioxide concentration is

$$C_{Arctic}\left(t_n\right) = 0.47 \sum_{i=1}^{n} E_{Arctic}\left(t_i\right)\left[0.0875\exp\left(\frac{t_n - t_i}{35600}\right) + 0.165\exp\left(\frac{t_n - t_i}{682}\right) + 0.733\exp\left(\frac{t_n - t_i}{50.8}\right)\right]$$

CO2 Concentration Contribution from Clathrates Release

So, the total carbon-dioxide concentration is now

$$C_{+Arctic}(t_n) = 280 + \frac{295-280}{2}\left[1 + \tanh\left(\frac{t_n - 1850}{34}\right)\right] + 1.14 C_{FF}(t_n) + C_{Arctic}(t_n)$$

Total CO2 Concentration

Legend: Total — Sum FF+NFF+Background — Permafrost ppmv

x-axis: year; y-axis: ppmv

The temperature would be

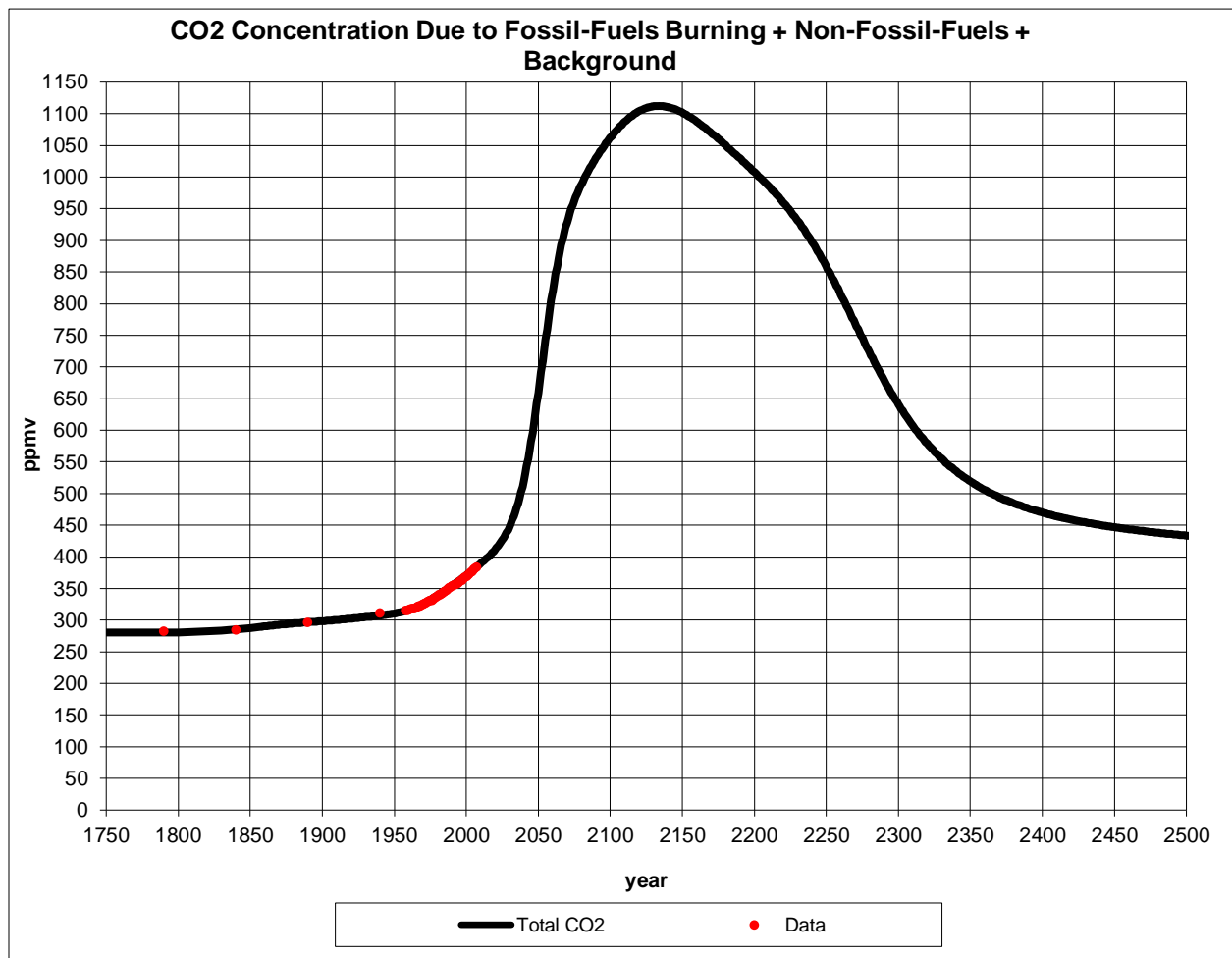$$dT_{+Arctic}\left(t_n\right) = \frac{3}{\ln 2}\sum_{i=1}^{n}\left\{0.368\tanh\left(\frac{t_n - t_i}{10.5}\right) + \frac{0.632}{2}\left[1 + \tanh\left(\frac{t_n - t_i - 277}{524}\right)\right]\ln\left(\frac{C_{+Arctic}\left(t_i\right)}{C_{+Arctic}\left(t_{i-1}\right)}\right)\right\}$$

Temperature Changed from 1700
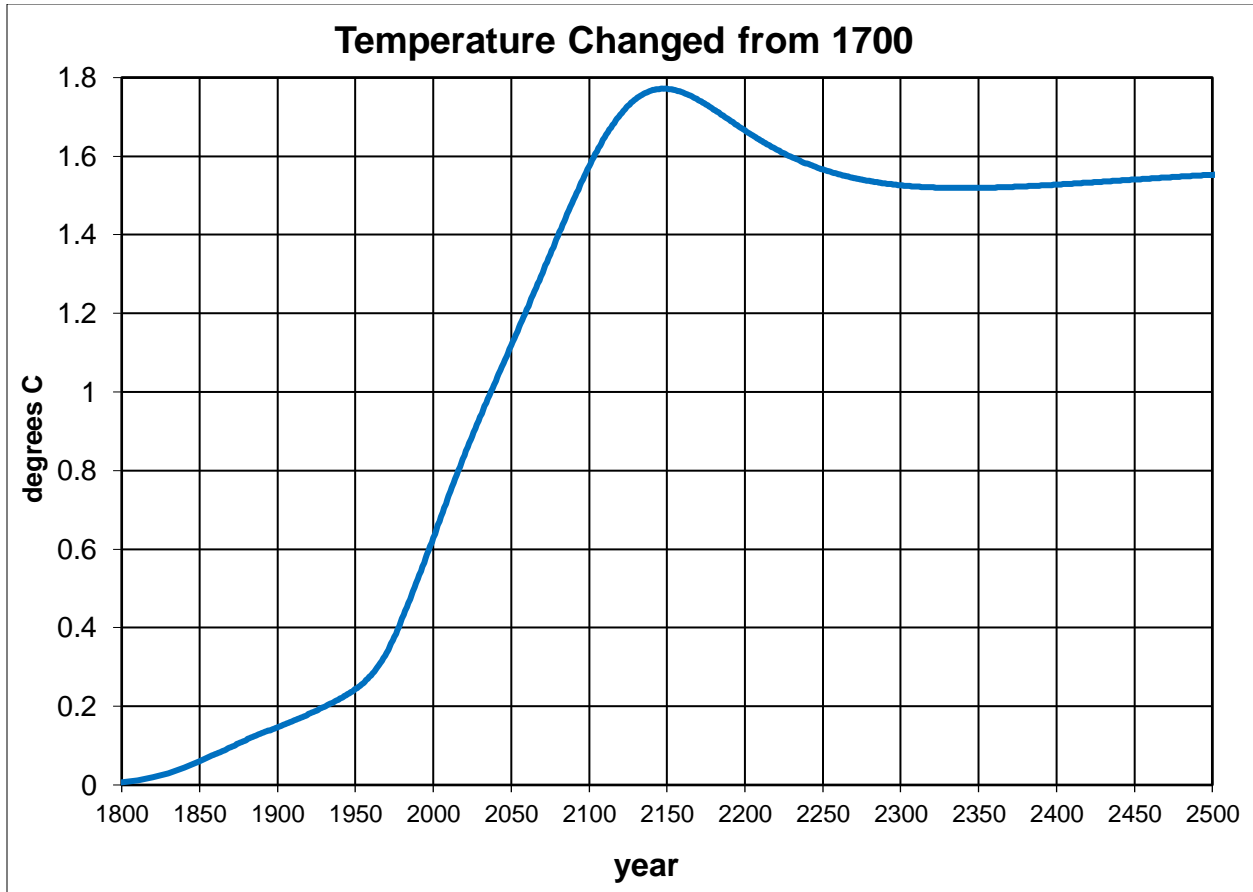
High Carbon-Dioxide Concentration Feedback, Arctic Carbon Release and Climate Sensitivity Increase (Worst Case)
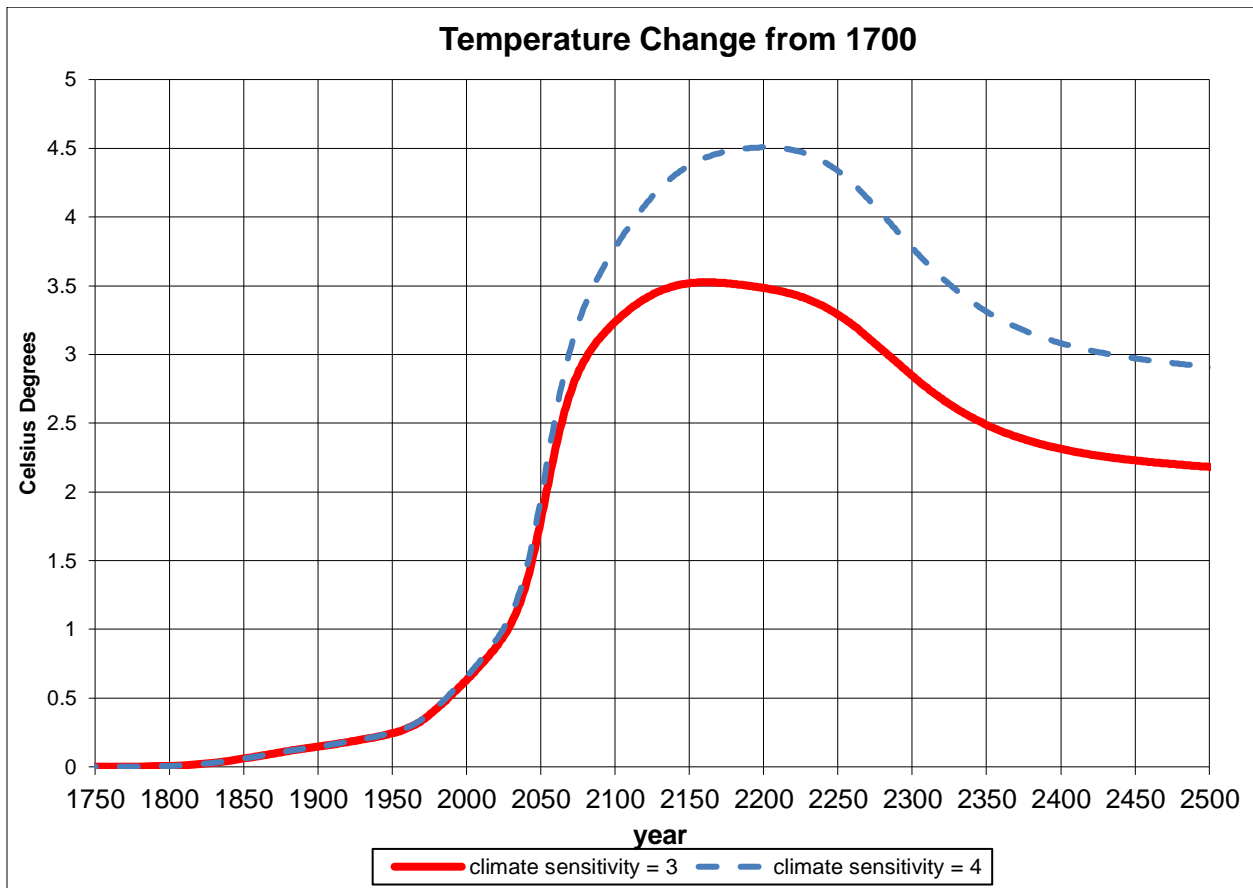
Suppose both Arctic carbon release and high carbon-dioxide concentration feedback on the concentration occurs:

$$C_{Both}(t_n) = \frac{C_{+Arctic}(t_n)}{2}\left[3 + \tanh\left(\frac{C_{+Arctic}(t_n) - 450}{25}\right)\right]$$

CO2 Concentration Due to Fossil-Fuels Burning + Non-Fossil-Fuels + Background

The temperature would be

$$dT_{Both}(t_n) = \frac{3}{\ln 2} \sum_{i=1}^{n} \left\{ 0.368 \tanh\left(\frac{t_n - t_i}{10.5}\right) + \frac{0.632}{2}\left[1 + \tanh\left(\frac{t_n - t_i - 277}{524}\right)\right] \ln\left(\frac{C_{Both}(t_i)}{C_{Both}(t_{i-1})}\right) \right\}$$

Temperature Change from 1700

The dashed curve is for a climate sensitivity changing from 3 to 4 by a hyperbolic tangent function:

$$dT_{BothTo4}(t_n) = \frac{1}{2\ln 2}\left[7 + \tanh\left(\frac{t_n - 2100}{50}\right)\right] \sum_{i=1}^{n} \left\{ 0.368 \tanh\left(\frac{t_n - t_i}{10.5}\right) + \frac{0.632}{2}\left[1 + \tanh\left(\frac{t_n - t_i - 277}{524}\right)\right] \ln\left(\frac{C_{Both}(t_i)}{C_{Both}(t_{i-1})}\right) \right\}$$