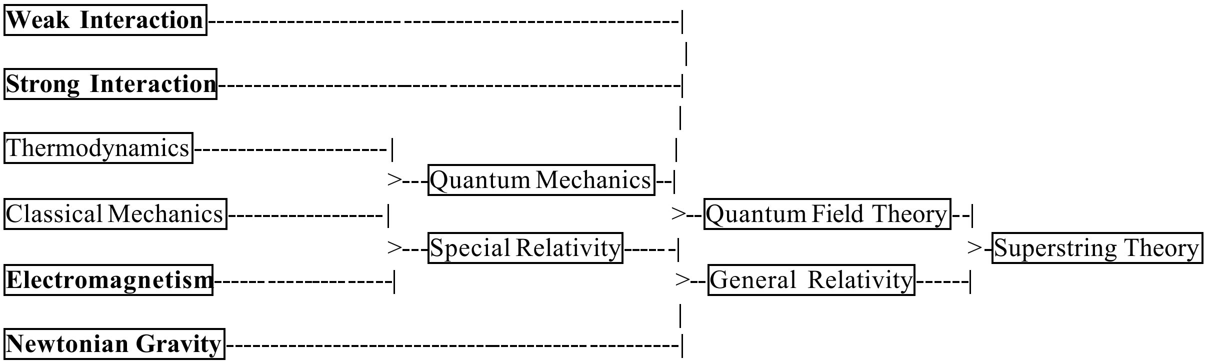
Summary of Theoretical Physics

L. David Roper, roperld@vt.edu

Written for tHe Book Group of Blacksburg VA
For the discussion of *The Elegant Universe* by Brian Greene, 10 October 2004
This file is available at http://www.roperld.com/physics/TheoreticalPhysicsSummary.rtf .

The following diagram summarizes the history of unification in Theoretical Physics:



This is not a perfect diagram. Electromagnetism (E&M) relativity concepts led Classical Mechanics to become Special Relativity, but Special Relativity does not explicitly include E&M. Also, E&M was not a part of Einstein's General Relativity until Kaluza added it in (see below). Special Relativity led Einstein to General Relativity and is included in Quantum Field Theory, but General Relativity is not included in Quantum Field Theory. The upper two and the lower two items (**bold**) on the left hand side are the four known interactions of physics. The other items, Thermodynamics, Classical Mechanics, Quantum Mechanics and Special Relativity are called dynamical theories, which apply to all interactions. Quantum Field Theory, General Relativity (and Kaluza's extension of it) and Superstring Theory combine dynamics with the four interactions.

There are four known interactions of particles in physics, in order of their discovery (See Table 2.):

- Gravitational Interaction (Newtonian Gravity -> General Relativity) (has infinite range) (only attractive) (mass serves as the "charge")
- Electromagnetism (E&M) (has infinite range) (electric charge)
- Weak (nuclear decays) Interaction (has small range of about 10^{-15} m) (2 weak charges)
- Strong (nuclear) Interaction, also called the Color Interaction (has small range of about 10^{-15} m) (3 strong charges)

Originally E&M was considered two separate interactions, Electricity and Magnetism. James Clerk Maxwell (1873) brought about the unification of Electricity and Magnetism with his set of equations, the first unification of interactions. (See the Appendix for some of the interactions' equations.) It turns out that the concepts of Einstein's Special Relativity theory are inherent in the Maxwell equations.

The principal concept of Special Relativity (Albert Einstein 1905) is that the speed of electromagnetic radiation (e.g., light), c, is the same for all observers in 3-D space. Another way to state it is that the speed of all objects is c in 4-D space-time. That is, an object at rest in 3-D space is moving at speed c in the time dimension. An object moving at speed $v < c$ in 3-D space, relative to some reference point, is moving at a speed less than c in the time dimension. An object with rest mass cannot move at a speed $v >= c$. Thus, the passage through the time dimension for two observers is different, depending on the relative speed of the two observers. Only a particle with rest mass zero can move at speed c in 3-D and it does not move at all in the time dimension. An infinite amount of energy is required to cause a particle with nonzero rest mass to move at speed c in 3-D.

The principal concept of General Relativity (Albert Einstein 1915) is that the Gravitation Interaction is equivalent to a warping of 4-D space-time. This hints that other interactions might also be related to the structure of space-time. The very weak Gravitation Interaction may or may not cause 3-D space to be closed. If it is closed, an object moving relative to very distant stars, with no sideways acceleration, will eventually return to its original location in 3-D space.

The principal concept of Quantum Mechanics (Erwin Schrödinger 1926) is that there are inherent, but calculable, uncertainties in physics, succinctly embodied in Heisenberg's Uncertainty Relation(Werner Heisenberg 1927): The position and momentum (mass times velocity) cannot be measured with arbitrary accuracy. (See below.)

Quantum Field Theory unified the Strong Interaction, the Weak Interaction and Electromagnetism with Quantum Mechanics and Special Relativity:

- The unification of E&M, Special Relativity, and Quantum Mechanics is called "Quantum Electrodynamics" or QED.
- The unification of the Weak Interaction with Quantum Electrodynamics is called the "Electroweak Theory".
- The unification of the Strong Interaction with the Electroweak Theory is called "Quantum Chromodynamics".
- Quantum Chromodynamics, along with the existence of the three families of "elementary particles", is called the "Standard Model". The Standard Model has 19 variable parameters that must be fixed by the known 12 particle masses and 7 interactions' strengths. (See Tables 1. and 2.)

At this point the only interaction lacking in the unification scheme was General Relativity. General Relativity posits that the structure of 4-D space-time is related to the structure of the gravitational interaction for point particles. All other theories before it just assumed the existence of space and time as their framework. So any unification of General Relativity (GR) with Quantum Field Theory (QFT) must include that concept.

The problem with unifying GR with QFT can be thought of as due to the Uncertainty Principle of Quantum Mechanics, which states that the position and the momentum (mass times velocity) cannot be measured at the same time to arbitrary accuracy. (The limit to the accuracy is given by Planck's constant: $h = 6.6262 \times 10^{-34}$ Joules-seconds, a very small number.) Thus, the concept of GR that the structure of space is intimately related to the Gravitational Interaction acting on a point particle is inconsistent with the Uncertainty Principle. String Theory implies that one cannot look at infinitesimal points in space, but only regions of space that are the size of the smallest string, which can be consistent with the Uncertainty Principle. In other words, String Theory drops the old requirement of point particles. The older theories' use of point particles works very well for spatial regions much larger than the sizes of the strings in String Theory.

Another historical unification that was important in eventually leading to Superstring Theory was the work of Theodor Kaluza (1919), who showed that, by introducing a fourth space dimension, the Maxwell E&M equations for Electromagnetism automatically emerge from the Einstein General Relativity (1915) equations. (See the Appendix for these equations.) Then Oskar Klein (1926) showed that the fourth space dimension should be very small (of the order of the Planck Length = 10^{-33} m = 10^{-18} x proton size), since the Electromagnetic Interaction strength is very large compared to the Gravitational Interaction. The structure of this extra dimension in 5-D space is intimately connected to the Electromagnetic Interaction, similar to the structure of 4-D space-time being intimately connected to the Gravitational Interaction. Klein connected the extra spatial dimension to the existence of electric charge.

String Theory (1970-85) was serendipitously discovered for the Strong Interaction by Gabriele Veneziano in 1968, although he did not recognize it as such. John Schwarz, Joseph Scherk and T. Yoneya showed that the theory was a string theory and that it contained a massless string vibration that had the correct spin (2 Planck units = $2\hbar = h/p$) for the Graviton, which is the messenger particle for the Gravitational Interaction. (See Table 2. for a list of the messenger particles for the four interactions.) So, this implied a possible unification of the Strong Interaction and the Gravitational Interaction, a step toward the long-sought goal initiated by Einstein with his search for a unified theory of all physics interactions. String

Theory contains a particle like the graviton, represents the Strong Interaction and satisfies Quantum Field theory. It appears to be the long-sought Unified Theory that Einstein spent his last years pursuing.

All of the theories until String Theory assumed that the "elementary particles" of Tables 1. and 2. for the Standard Model are point particles; i.e., that they have no extension in space. String Theory replaces point particles by very small, but finite length, strings with zero thickness. In String Theory the particles and all their properties are determined by the mathematics of the vibrations of strings in multi-dimensional space. The mass of a particle is given by the energy (using Einstein's equation $m = \frac{E}{c^2}$) of the string vibration. The strength of one of the four interactions is proportional to the length of the string that represents the corresponding messenger particle (see Table 2.) for that interaction.

The five original versions of String Theory had 9 space dimensions and 1 time dimension, because of stringent conditions of physical logic. The 6 extra spatial dimensions beyond 3 had to exist to satisfy stringent physical conditions. It was later shown by Edward Witten (1995) that those five different versions of String Theory were just different ways to mathematically describe the more general Superstring "M Theory" for certain special cases. Superstring M Theory requires that there be 10 space dimensions and 1 time dimension. The exact equations for Superstring M theory are not known; only approximate equations are available for calculations at present. Even those approximate equations cannot be solved exactly. Instead, approximate methods of solving them must be used. Superstring Theory has **only one variable parameter** (the tension of the string or "coupling constant", it has no mass) instead of 19 variables (12 particle masses and 7 interaction strengths of particles) as in the Standard Model.

The exact shape of the small extra 7 spatial dimensions is very important to yield the correct physics. For example, the number of holes in them determines the number of particle families in the Standard Model. Table 1. shows the three known families, which indicates that there should be three holes in the 7 spatial dimensions. However, future experiments may find more families at higher energies.

Similar to the concept that the structure of 4-D space-time is intimately connected to the Gravitational Interaction in General Relativity, it appears that the structure of 11-D spacetime is intimately connected to all physical interactions: Gravitational, Weak, Electromagnetic and Strong; and perhaps others even weaker than the Gravitational Interaction that have not yet been discovered, which could be associated with even more spatial dimensions.

Considering the Kaluza-Klein theory combining the Gravitational and Electromagnetic Interactions described above, one could argue that each interaction charge (coupling constant) other than the Gravitational Interaction requires an extra space dimension. Using the same argument as Klein used for the size of the extra spatial dimension connected to the electric charge, one gets sizes for the extra spatial dimension due to the **two** Weak Interaction charges as 10^{-24} m and for the **three** Strong Interaction charges as 10^{-37} m. Thus, we have 4-D space-time due to the Gravitational Interaction, plus 1-D due to the E&M Interaction, plus 2-D due to the Weak Interaction and plus 3-D due to the Strong Interaction, for a total of 10 dimensions. The 11th dimension is the "weird" dimension that unites the original string theories into the Superstring M Theory. (See below for more about the 11th dimension.)

The Supersymmetry aspect of Superstrings predicts that all of the particles in the Standard Model have much more massive (about 20 times the proton mass) partners whose spins differ by a $\frac{\hbar}{2}$ unit of spin from their partner particles in the Standard Model. This is the **major prediction of Superstring M Theory to date**. Perhaps the Large Hadron Collider being built in Europe will be able to find these super-particles when it starts running in 2007.

Superstring M theory indicates that 12 of the interactions' 13 messenger particles (8 gluons, photon, 2 Ws and the Z; see Table 2.), excluding the graviton, are linear strings, not a loop string. The graviton is a string loop. This is an important distinction that will be used below.

Superstring M Theory allows for the possibility of having not only vibrating strings, but also vibrating membranes in 2 dimension, vibrating bodies in 3 dimensions and similar vibrating objects in higher space dimensions. A point is now called a 0-brane, a string is called a 1-brane, a membrane is called a 2-brane and an arbitrary space dimension (p) is called a p-brane. Generally, the higher the order of a brane, the larger the mass for a characteristic length and vibration of the brane. (For example, in acoustics, a vibrating circular drum membrane of diameter L can be considered as made of many strings of length L attached to each other spanning the circular membrane.) So strings (1-branes) may be the only branes that need to be considered for low-energy calculations, which includes most of the world observable by humans.

It has been shown from the mathematics that lower-dimensional branes within a higher-dimensional brane attach themselves strongly to the enclosing brane; this is called "sticky branes". This property has led to a brane theory of the universe, as follows:

It has been proposed that the 3-D universe space is a large 3-brane with strings and smaller-dimensional branes inside it and attached to it (a "braneworld"). Superstring M Theory indicates that all of the interaction messenger particles other than the graviton (a string loop with no ends to attach) have both ends of their strings attached to this universe-defining 3-brane. Thus, only the graviton can communicate with the other 7 spatial dimensions. Given this braneworld theory of the universe, the extreme weakness of the Gravitational Interaction could be because it is actually a much stronger interaction, but it is made weak in the braneworld because some of the other 7 spatial dimensions are not extremely small and, thus, they take up a large part of the interaction. That is, since the strength of the interaction is proportional to the length of the messenger-particle string, some of the length of the graviton is not in the braneworld. So our braneworld universe sees only a small part of the Graviton string length, which means that the Gravitational Interaction is very weak. See the book *The Fabric of the Cosmos: Space, Time and the Texture of Reality* by Brian Greene, 2004 for more information about braneworlds.

Only three of the 10 space dimensions are measurable by current instruments. The number of observable space dimensions is revealed by the spatial dependence of the Gravitational Interaction. It is $1/r^2$ (Newton's law of gravity) for 3 spatial dimensions and $1/r^{p-1}$ for p spatial dimensions. The dependence $1/r^2$ has been verified only down to about 0.1 millimeters, so some or all of the extra 7 spatial dimensions could be almost that large.

A bedrock principal of physics is the conservation of energy/mass, which has never been shown to be violated. (The Heisenberg Uncertainty Principle in Quantum Mechanics allows non-conservation of energy for very short periods of time, but not for long periods of time.) However, since gravitons can "leak" into the extra spatial dimension, a small amount of energy could "disappear" from 3-D space. This might be detectable in future experiments. Then, although energy would be conserved in the entire 10-D space, it might not be completely conserved in 3-D space.

I find the 10th spatial dimension especially mysterious, since it only appears in the generalization of the original 5 String Theories to the Superstring M Theory. In the book *The Fabric of the Cosmos: Space, Time and the Texture of Reality* by Brian Greene, one of the space dimensions is considered as a possible connection between different braneworld universes to create the Big Bang as a cyclic phenomenon as the strings connecting the two braneworlds through the 7th space dimension pull the two braneworlds into cyclic collisions. My thoughts: It could be that one of the two tenuously connected braneworlds is made of matter and the other is made of anti-matter. Then when they collide, the Big Bang could be a massive matter/anti-matter annihilation.

The Particles of the Standard Model

Table 1. 12 Elementary Particles in the Standard Model (masses in units of the proton mass; all have spin $\frac{\hbar}{2}$):

Type	Electric Charge	Baryon Charge	Lepton Charge	Family 1		Family 2		Family 3	
				Particle	Mass	Particle	Mass	Particle	Mass
Lepton	-1	0	+1	Electron	0.00054	Muon	0.11	Tau	1.9
Lepton-Neutrino	0	0	+1	Electron-neutrino	$< 10^{-9}$	Muon-neutrino	$< 10^{-4}$	Tau-neutrino	$< 10^{-3}$
Quark 1	$+\frac{2}{3}$	+1	0	Up-quark	0.0047	Charm-quark	1.6	Top-quark	189
Quark 2	$-\frac{1}{3}$	+1	0	Down-quark	0.0074	Strange-quark	0.16	Bottom-quark	5.2

Each of these particles has an anti-particle with opposite electric charge, opposite baryon “charge” and opposite lepton “charge”, but the same mass and spin. (For example, the Anti-Electron has electric charge +1, lepton charge -1, baryon charge 0 and is called the “Positron”.) Electric charge, lepton charge, and baryon charge are always conserved in all four particle interactions. Our universe is made of matter; antimatter is only produced for short periods of time, because it annihilates with matter when they interact.

Combinations of Quarks and Anti-Quarks make up “Hadrons”, the particles that participate in the Strong Interaction, such as the Proton and Neutron and p Mesons. The Leptons and Anti-Leptons do not participate in the Strong Interaction; i.e., only baryon-charged particles (Quarks, Anti-Quarks and Hadrons constructed from Quarks and Anti-Quarks) participate in the Strong Interaction. Only the electric-charged particles participate in the Electromagnetic Interaction. All 12 of these particles participate in the Weak Interaction and the Gravitational Interaction. (Even if the neutrinos turn out to be “massless at rest”, they interact gravitationally because they always move at the speed of light and carry energy E and, therefore, have a “dynamical” mass given by Einstein’s equation $m = \frac{E}{c^2}$. Even photons and gravitons have a “dynamical” mass.)

Table 2. The four interactions and their messenger particles (13 of them):

Interaction	Relative Strength of Interaction	Messenger Particle (All have 0 electric charge, unless otherwise indicated by a superscript.)	Mass of Messenger Particle (units of proton mass)	Spin of Messenger Particle (units of $\hbar$)
Strong (3 strong charges: r, g, & b; r+g+b=0)	1	8 Gluons (strong charged)	0	1
Electromagnetic (1 electric charge: plus+minus=0)	10^{-2}	Photon	0	1
Weak (2 weak charges)	10^{-13}	W^+, W^- and Z	86, 97	1
Gravity (1 gravity charge)	10^{-39}	Graviton	0	2

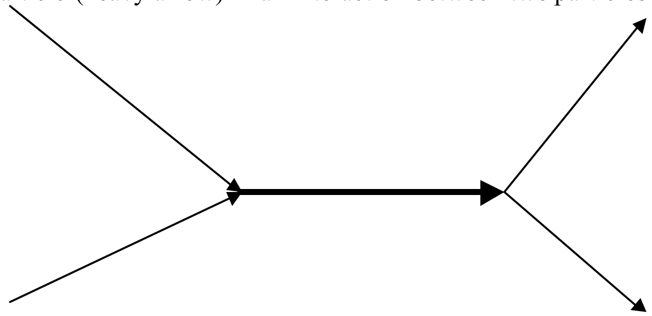
The 8 Gluons mediate the Strong Interactions of Quarks. The Photon mediates the E&M Interaction for the electrically-charged particles of Table 1. only. The other 4 messenger particles mediate the Weak and Gravitational Interactions for all 12 particles in Table 1. Since the Gluons, the Photon and the Graviton have zero rest mass, they always move at the speed of light. The Graviton has not been “seen” yet, because the Gravitational Interaction is so weak; massive experiments have been and are searching for gravitational waves. The baryon charge in Table 1. is related to the 3 strong charges in Table 2. and the lepton charge in Table 2. is related to the 2 weak charges in Table 1.

The number of arbitrary parameters in the Standard Model = 12 particles and 7 charges = 19.

Since neutrinos have no electric charge and no strong charge, they interact only by the Weak and the Gravitational Interactions. These interactions are so weak that huge numbers of neutrinos from outer space pass through a human body and the earth every second without interacting. (Would an advanced civilization in outer space communicate across the universe solely by using neutrinos instead of photons, to insure that only other similarly advanced civilizations would be able to decipher the messages? Would the most advanced ones communicate by using gravitons, since they are much more difficult to detect than are neutrinos?!)

An interesting thing is that all 13 of the messenger particles, including the Graviton, interact by the Gravitational Interaction; i.e., with the Graviton as the messenger particle. That is, the Gravitational Interaction is of very special importance, since it determines the structure of the 4-dimensional space-time.

Role of a messenger particle (heavy arrow) in an interaction between two particles (light arrows):



Appendix: Some Physics Interactions' Equations

The following are the **Maxwell equations for Electromagnetism**:

$$\begin{aligned}
\nabla \cdot \mathbf{E} &= 4\pi\rho \\
\nabla \times \mathbf{E} &= -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \cdot \mathbf{B} &= 0 \\
\nabla \times \mathbf{B} &= \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t},
\end{aligned}$$

E is the vector electric field in 3-D space and **B** is the vector magnetic field in 3-D space. Here ? is the charge density, **J** is the vector current density and *c* is the speed of light.

These equations are deceptively simple looking. However, $\nabla \cdot E$ and $\nabla \times E$, for example, are complicated vector operations involving the three coordinates in 3-D space.

The **Einstein equations for General Relativity** are even simpler looking: $G_{mn} = 8pGT_{mn}$
However, the subscripted *G* and *T* are complicated mathematical tensor operators. The unsubcripted *G* is the gravitational strength. If an extra spatial dimension is added (4-D instead of 3-D) is added, the Maxwell E&M equations are included in this equation.

Newtonian gravity is a special case of the General Relativity equations:

$$\begin{aligned}
\nabla \times \mathbf{g} &= \mathbf{0} \\
\nabla \cdot \mathbf{g} &= -4\pi G\rho,
\end{aligned}$$

where ? is the mass density and **g** is the gravitational field vector.

The following is the **Schrödinger equation for non-relativistic Quantum Mechanics**:

$$i\hbar\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi\ .$$

Here ? is the wave function from which probabilities are calculated, V is the energy potential in which a particle of mass *m* is located and ∇^2 is a complicated calculus operator in 3-D space. The symbol *i* is the imaginary number $\sqrt{-1}$ and $\hbar$ is Planck's constant *h* divided by 2p.

There are relativistic versions of quantum mechanics represented by the **Klein-Gordon equation** for spin-0 particles, such as the p meson, and the **Dirac equation** for spin-$\frac{\hbar}{2}$ particles, such as the electron and the proton. The introduction of Special Relativity to Quantum Mechanics automatically introduced spin.

The **Heisenberg Uncertainty Relation** is $\Delta x \Delta p \geq \hbar / 2$, where x = position and p = momentum = mv, where m = mass.