The linear equation for the number of senators for a state is

United States Senate According to the Colonial Population

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3 February 2019

Introduction

The Constitution of the United States, ratified in 1788, states in Article I Section 3 1.:

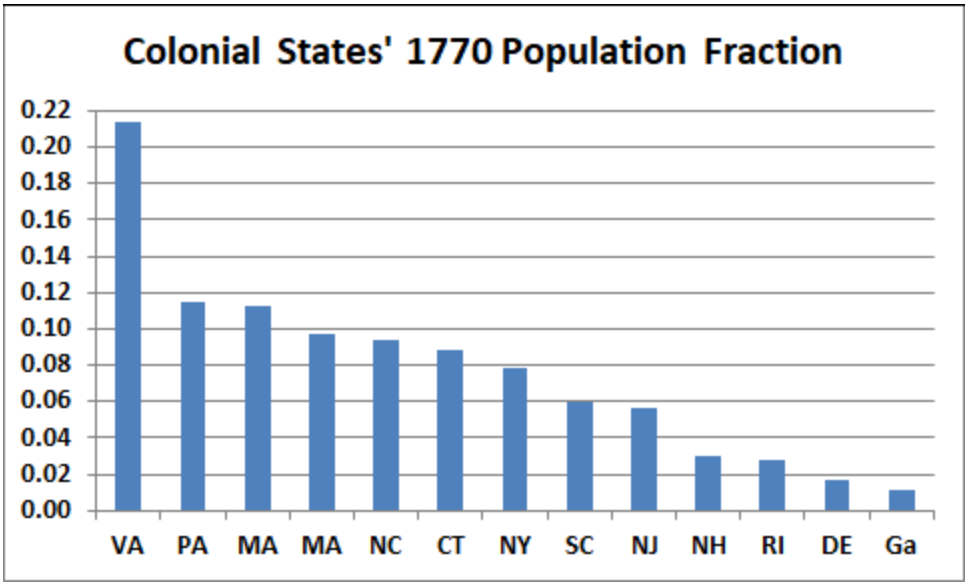
" The Senate of the United States shall be composed of two Senators from each State, chosen by the Legislature thereof,[3] for six Years; and each Senator shall have one Vote."

In Article XVIII 1.:

" The Senate of the United States shall be composed of two Senators from each State, elected by the people thereof, for six years; and each Senator shall have one vote. The electors in each State shall have the qualifications requisite for electors of the most numerous branch of the State legislatures."

The estimated populations of the thirteen colonies in 1770 are:

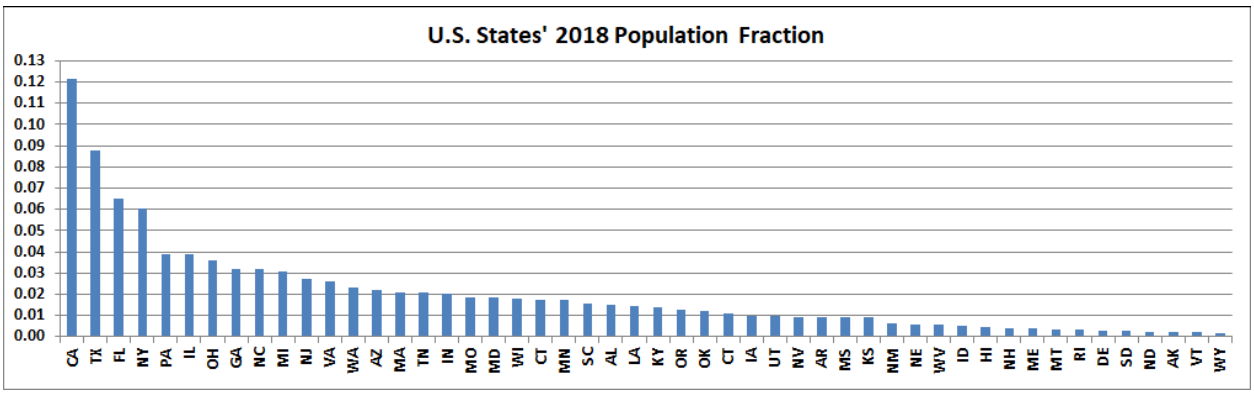
State	Population	Fraction
VA	447,016	0.214
PA	240,057	0.115
MA	234,808	0.112
MD	202,599	0.097
NC	197,200	0.094
CT	183,881	0.088
NY	162,920	0.078
SC	124,244	0.059
NJ	117,431	0.056
NH	62,396	0.030
RI	58,196	0.028
DE	35,496	0.017
Ga	23,375	0.011
Sum	**2,089,619**	**1**



The ratio of highest state population fraction to lowest is 19.12368 or about 20. **It is reasonable to assume that the intent of the signers of the Constitution was to deviate the Senate from democracy by a factor of 20.** Call this 20 factor the "**Anti-Democracy Ratio = ADR**". How does this 1770 deviation [ADR(1770)] compare to the 2018 deviation [ADR(2018)]?

2018 Population of the States in the United States

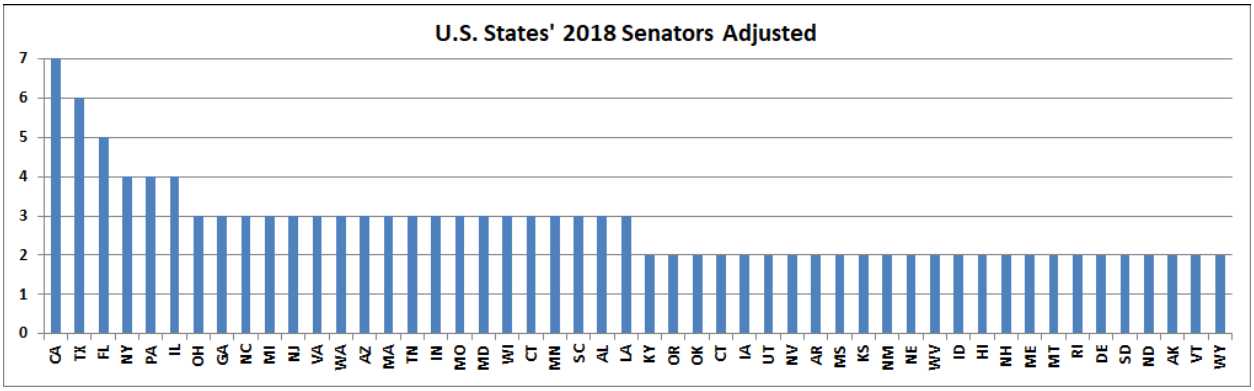
The population fractions of the states in the United States in year 2018 are:



The ratio of highest to lowest is 71.35294 or about 70, the **2018 Anti-Democracy Ratio [ADR(2018) = 70] for the Senate**. **The ratio ADR(2018)/ADR(1770) = 70/20 = 3.5.**

Linearly Adjusted Senators for States

To make a 2018 adjusted Anti-Democracy Ratio [ADR(2018) = 70] for senators allotted to states match the Anti-Democracy Ratio when the constitution was ratified [ADR(1770) = 20], the most populous state (California) should have 3.5 more senators than the least populous state (Wyoming). Of course, the number of senators for each state by this calculation must be rounded to an integer. If every state should have at least two senators, the total number of senators would be 137.

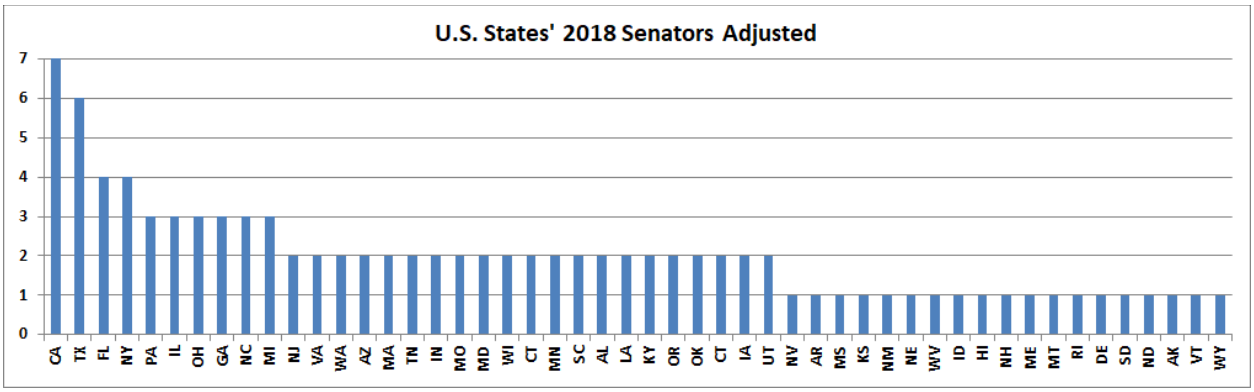


The linear equation for the number of senators for a state is

$$N(state) = Integer\left[\frac{2(3.5-1)}{f_{max}-f_{min}}(f-f_{min})+2\right].$$

The total number of senators would change when the state fractions f change. It would be reasonable to calculate the state fractions every 10 years and, thereby, change the distribution of senators.

To restrict the total number of senators to 100:



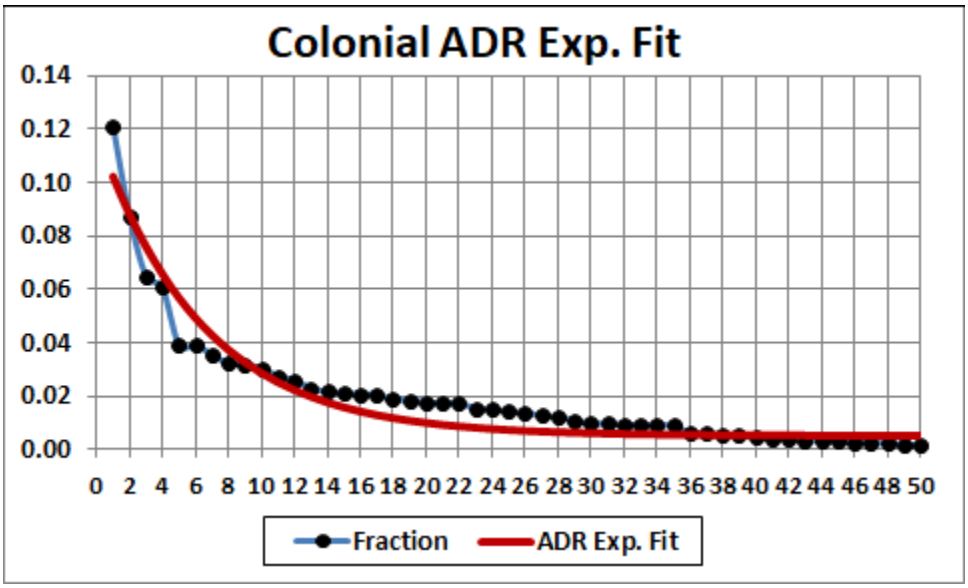
The linear equation for the number of senators for a state is

$$N(state) = Integer\left\{1.065\left[\frac{3.5-1}{f_{max}-f_{min}}(f-f_{min})+1\right]\right\}.$$

The factor 1.065 would have to be determined such that the sum over the states = 100 when the state fractions f change.

Exponentially Adjusted Senators for States

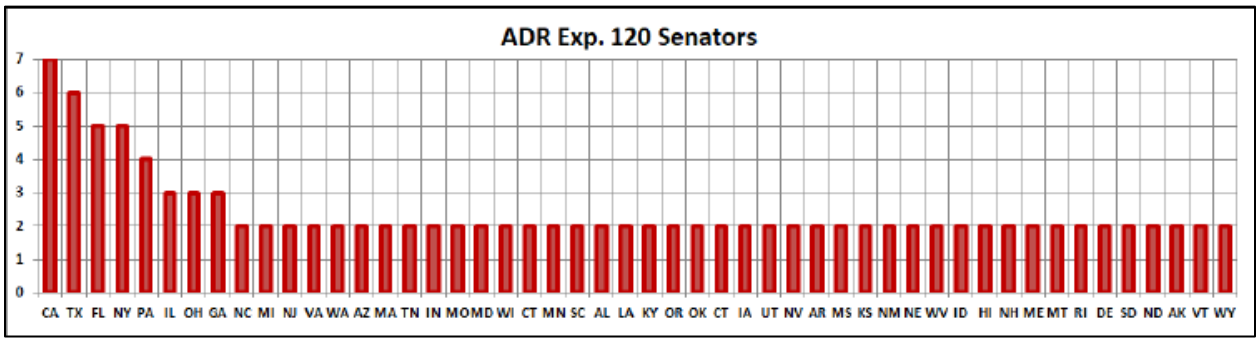
The calculation above used a linear function to represent the population versus ranked states, which is a poor fit to the data. A better fit to the data is an exponential fit. Here is an exponential fit to the population data such that the ADR is equal to 20, the colonial 1770 value:



The exponential fit function is

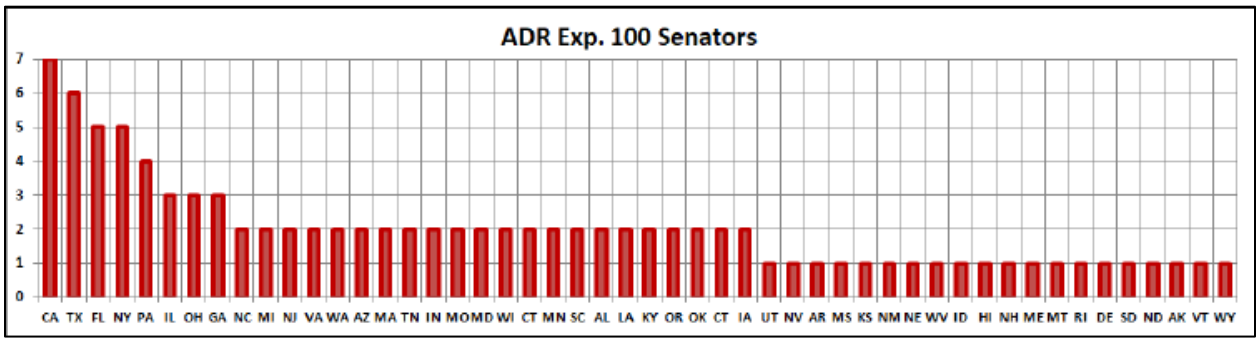
$$\boxed{f\left(Nstate\right)=0.00507+0.1132\exp\left[-\left(N-0.03075\right)/6.349\right]}\ \text{where } N=\text{integer from 0 to 50.}$$

For a minimum senator number of 2 per state this exponential function yields the following senators/state allocation:



The total number of senators is 120.

In order to have only 100 senators the following distribution is needed:



Electoral College

What about the Electoral College? Currently the ratio of largest to smallest is about 18, far below the 71 ratio of population. The proposed change described above for 100 senators would make the Electoral-College ratio be about 30, much closer to the population ratio, and the change proposed above for 120 senators would make the Electoral-College ratio be about 20, not much different than the population ratio.

Conclusion

This change in the Constitution has a negligible chance of occurring. However, if the disparity in population among the states continues as it has since colonial days, the stability of the United States might require such a change.

Ideally the constitutional change would include a procedure for allotting the number of senators for each state according to the population distribution every ten years.

At the rate of Anti-Democracy-Ratio change of 50/248-years = 0.202 per year since colonial days, in 2100 the Anti-Democracy Ratio would be ADR(2100) = 70 + 0.202/year x 82-years = 87. At that rate of increase the ADR would be 100 in 149 years, year 2167 [ADR(2167) = 100].

References

- The Senate Problem
- Our Senate Problem
- America has a Big Senate Problem, and It's Only Getting Worse
- So you want to change the Senate?