Log-Verhulst Function at Large x

The Log-Verhulst function (http://www.roperld.com/economics/IncomeDistribution.htm#A2P3) is

$$\boxed{p(x) = \frac{(2^n-1)\left(\frac{x}{X}\right)^{\frac{1}{w}}}{nwx\left[1+(2^n-1)\left(\frac{x}{X}\right)^{\frac{1}{w}}\right]^{\frac{n+1}{n}}}}.$$

It is desired to know the behavior of $p(x)$ as $x \to \infty$.

$$\lim_{x\to\infty} p(x) = \lim_{x\to\infty} \frac{(2^n-1)\left(\frac{x}{X}\right)^{\frac{1}{w}}}{nwx\left[1+(2^n-1)\left(\frac{x}{X}\right)^{\frac{1}{w}}\right]^{\frac{n+1}{n}}} = \frac{(2^n-1)\left(\frac{x}{X}\right)^{\frac{1}{w}}}{nwx\left[(2^n-1)\left(\frac{x}{X}\right)^{\frac{1}{w}}\right]^{\frac{n+1}{n}}} =$$

$$\frac{1}{nwx}(2^n-1)\left(\frac{x}{X}\right)^{\frac{1}{w}}\left((2^n-1)\left(\frac{x}{X}\right)^{\frac{1}{w}}\right)^{\frac{1}{n}(-n-1)} = \frac{(2^n-1)^{\frac{-1}{n}}}{nwx}\left[\left(\frac{x}{X}\right)^{\frac{1}{w}}\right]^{\frac{-1}{n}} =$$

$$\frac{1}{nw(2^n-1)^{\frac{1}{n}}}x^{-1}\left[\left(\frac{x}{X}\right)^{\frac{1}{w}}\right]^{\frac{-1}{n}} = \frac{(X)^{\frac{1}{nw}}}{nw(2^n-1)^{\frac{1}{n}}}x^{-1-\frac{1}{nw}} = \frac{(X)^{\frac{1}{nw}}}{nw(2^n-1)^{\frac{1}{n}}}x^{-\left(\frac{nw+1}{nw}\right)}$$

because $1 + \frac{1}{n}(-n-1) = -n^{-1}$.

Inverse power of x, P, is given by $-1 - \frac{1}{nw} \equiv -P$.

$\therefore$ $\boxed{P = 1 + \frac{1}{nw} \text{ or } nw = \frac{1}{P-1}}$.